1. (BFS, 3.6) Consider the model of a covalent modification cycle from Chapter 2 of BFS, with the additional requirement that the protein kinase $Z$ is produced and decays according to the reaction below:

$$Z + X \rightarrow_{k_f} C \rightarrow_{k_{cat}} X^* + Z, C \rightarrow_{k_r} Z + X$$

$$Y + X^* \rightarrow_{k'_f} C' \rightarrow_{k'_{cat}} X + Y, C' \rightarrow_{k'_r} Y + X^*$$

$$Z \rightarrow_{\delta} \emptyset, \emptyset \rightarrow_{u(t)} Z$$

and take $k_r, k_f \gg k_{cat}, \delta, u(t)$.

Employ singular perturbation with small parameter $\epsilon = \frac{\delta}{k_r}$ to obtain the approximated dynamics of $Z(t)$ and $X^*(t)$.

2. In this problem, you will simulate the Notch Delta dynamics of fine patterning as a cellular automaton coupled with continuous dynamics, inspired by Collier’s paper reviewed in class. The code is provided, you will just need to add code where specified.