CALIFORNIA INSTITUTE OF TECHNOLOGY

ME/CS 132b, Spring 2011

Problem Set #1 Due: Apr 7th, 2011

Readings: LaValle, Chapter 1 & 3 (all), Chapter 13 (Sections 13.1, 13.2, and 13.3 only).

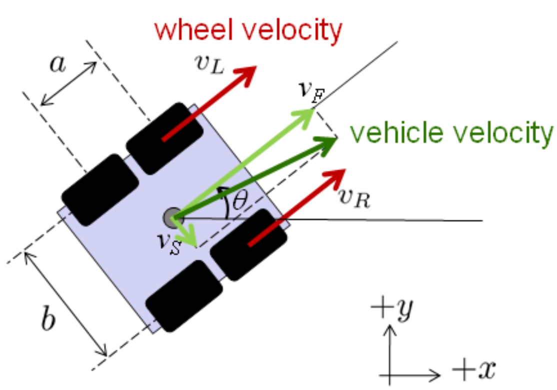
1. $[3+3+3+5+5+6 = 25$ points$]$ LaValle, Problems 1–5 and 7 of Chapter 13 (6 problems in total).

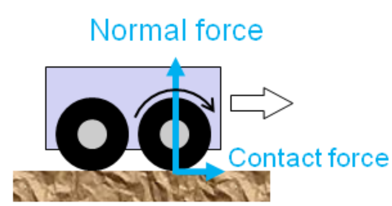
   Note: In equation (13.19) of the book, the equation for $\dot{\theta}_1$ should be

   $$\dot{\theta}_1 = \frac{s}{d_1} \sin(\theta_0 - \theta_1).$$

2. [25 points] In this problem, you will be asked to derive the state-space model of a 4-wheel skid-steered vehicle. Please use the following notations:

   - a: wheelbase
   - b: wheel track
   - v_L: circumferential velocity of left wheels (in body frame)
   - v_R: circumferential velocity of right wheels (in body frame)
   - v_F: forward speed of the vehicle (in body frame)
   - v_S: lateral speed of the vehicle (in body frame)
   - m: vehicle mass
   - J: moment of inertia around C.G. (assume C.G. is at the vehicle center)
   - x, y: vehicle position
   - θ: vehicle heading
   - C: tire stiffness
   - μ_0: friction coefficient





   (a) The vehicle is driven by the friction forces generated from the wheels. Let F_{FL}, F_{FR}, F_{RL}, and F_{RR} denote the longitudinal friction forces acting on the four tires, respectively (the first subscript denotes front/rear and the second denotes left/right), and S_{FL}, S_{FR}, S_{RL}, and S_{RR} denote the lateral forces. Assume all the F's and S's are the control inputs, and $(x, y, \theta, \dot{x}, \dot{y}, \dot{\theta})$ are the states, derive the state-space model from the equation of motion.

(b) In practice, we cannot apply the friction forces directly. Instead, the forces are generated by applying velocities on the wheels, of which we have direct control. For simplicity, assume that both left wheels have velocity v_L, and both right wheels have velocity v_R. Assume that v_L and v_R are the control inputs, and $(x, y, \theta, \dot{x}, \dot{y}, \dot{\theta})$ are the states, derive the new state-space model.

When deriving the friction forces, use a simple friction model: $P = Cv_{\text{slip}}$ when $Cv_{\text{slip}} \leq \mu_0 W$ and $P = \mu_0 W$ otherwise, where v_{slip} is the slip velocity, and W is the weight acting on the ground contact point of the tire. Note that each tire slips laterally due to v_S and skids longitudinally due to the difference between v_F and v_L (or v_R). The total slip v_{slip} at each tire consists of the lateral and longitudinal slips. Also note that the friction vector points in the opposite direction of the slip velocity.

3. [20 points] Implement a dynamics model of a skid-steered vehicle in MATLAB (or any other programming languages of your choice). The function should look like:

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[dxdt] = dynamics_skid_steer(x, u)
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4. [15 points] Implement an Euler integrator and integrate this with the model developed in Problem #3 in MATLAB (if you have trouble solving #3, take any model covered in the lecture). Use the following parameters:

Parameter	Value	Unit
a	0.4	m
b	0.5	m
m	8	kg
J	2	$\text{kg} \cdot \text{m}^2$
C	5000	N/(m/s)
μ_0	0.4	(no unit)
g	9.8	m/s^2

Run the simulation with four different integration step sizes dt: 0.01 sec, 0.1 sec, 1.0 sec, and 2.0 sec, and plot the vehicle's states (x, y, θ etc.) as a function of time, in a separate 2D plot. Start from the origin (all states are zeros), and use the following control input:

$$v_L = \begin{cases} 0.1t & 0 \leq t \leq 20, \\ 0 & t > 20, \end{cases}$$

$$v_R = \begin{cases} 0.2t & 0 \leq t \leq 10, \\ 2 & 10 < t \leq 20, \\ 0 & t > 20. \end{cases}$$

5. [15 points] Implement the 4th-order Runge-Kutta integration scheme. Run the simulation with the same setup used in Problem #4 *and* compare the results.