CALIFORNIA INSTITUTE OF TECHNOLOGY
Computing and Mathematical Sciences

CS 142

R. Murray, M. Chandy

Homework Set #1

Fall 2019

Issued: 2 Oct 2019
Due: 9 Oct 2019

Note: In the upper left hand corner of the *second* page of your homework set, please put the number of hours that you spent on this homework set (including reading).

Notes on homework for CS 142:

- Please turn in your homework sets using Moodle. You do not need to typeset your solutions; a scanned copy of handwritten solutions is fine.
- You get two grace periods of no more than 2 days each for late homework. After that, late homework will not be accepted without a note from the dean or the health center.
- Solutions to the homework sets will be made available on the day that the homework sets are handed back.

Homework problems for this week:

1. Prove the following:

   (a) The distribution law:

   $$[X \vee (Y \wedge Z) \quad \equiv \quad (X \vee Y) \wedge (X \vee Z)].$$

   (b) The absorption law:

   $$[X \wedge (X \vee Y) \quad \equiv \quad X].$$

2. Prove Theorem 5 in Sivilotti: $[X \implies Y \quad \equiv \quad \neg X \vee Y]$.

3. The predicate $x \not\equiv y$ holds if **exactly one** of the predicates x and y is *true*. The proof that $\not\equiv$ is commutative is straightforward.

   (a) Show that $\not\equiv$ is also associative:

   $$(x \not\equiv y) \not\equiv z \;\equiv\; x \not\equiv (y \not\equiv z)$$

   (b) Consider a predicate X defined as $x[0] \not\equiv x[1] \not\equiv \ldots \not\equiv x[n]$. Which of the following statements are true?

      i. X is *true* if and only if the number of predicates $x[i]$ that evaluate to *true* in the expression is an even number.

      ii. X is *true* if and only if the number of predicates $x[i]$ that evaluate to *false* in the expression is an even number.

      iii. X is *true* if and only if the number of predicates $x[i]$ that evaluate to *true* in the expression is an odd number.

iv. X is *true* if and only if the number of predicates $x[i]$ that evaluate to *false* in the expression is an odd number.

v. None of the above.

(c) Prove your answer to problem 3b.

4. For a set X, the notation

$$\forall x \in X : p(x) : q(x)$$

stands for

for all x in X where $p(x)$ holds, $q(x)$ is *true*

or, equivalently,

for all x in X, $p(x) \Rightarrow q(x)$,

where $\Rightarrow$ is implication. The set X is often evident from the context and is left out of the formula, as in:

$$\forall x : p(x) : q(x),$$

where, for example, the set X is the state of a system, or numbers, or the set of students at Caltech, depending on the context of the statement.

Note: In the course textbook, $p(x)$ is written as $p.x$, but we use the functional form to emphasize that p is a predicate—a function from some space to the Boolean constants.

Likewise

$$\exists x \in X : p(x) : q(x)$$

stands for

there exists an (i.e. at least one) x in X where $p(x)$ holds for which $q(x)$ is *true*

or equivalently

there exists an x in X for which $p(x)$ and $q(x)$ hold,

which we write mathematically as

$$\exists x \in X : p(x) \wedge q(x)$$

Again, when the set X is obvious from the context we write:

$$\exists x : p(x) : q(x)$$

(a) Is the following statement true?

$$\neg(\forall x : p(x) : q(x)) \;\equiv\; (\exists x : p(x) : \neg q(x))$$

(b) Prove your answer for a finite set X or give a counter example. Recall that if X is $\{x_0, \ldots, x_n\}$ then:

$$\begin{aligned} \forall x \in X : r(x) \;&\equiv\; r(x_0) \wedge r(x_1) \wedge \ldots \wedge r(x_n) \\ \exists x \in X : r(x) \;&\equiv\; r(x_0) \vee r(x_1) \vee \ldots \vee r(x_n) \end{aligned}$$

5. Consider the following program:

$$
\begin{array}{ll}
\textbf{Program} & \textit{Sivilotti-2.5.2} \\
\textbf{var} & b : \text{Boolean} \\
 & n : \{0, 1, 2, 3\} \\
\textbf{initially} & n = 1 \\
\textbf{assign} & \\
 & \quad b = 1 \rightarrow n := n +_4 1 \\
 & [\!] \;\; n = 0 \rightarrow b := \textbf{false}
\end{array}
$$

The addition in the first assignment is performed modulo 4.

(a) Draw a directed graph that represents this program. Make sure to label the initial nodes and label all actions using the action numbers above.

(b) Show that under the assumption of week fairness there is no guarantee that this program reaches a fixed point. Is there a fixed point under the assumption of strong fairness? If so, what is it?

(c) Does this program terminate?