CALIFORNIA INSTITUTE OF TECHNOLOGY
Control and Dynamical Systems

CDS 102
Problem Set #1

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Reading:

Abraham, Marsden, and Ratiu, Ch 1.

Optional: For a review of basic set operations see Halmos, *Naive Set Theory*. For a more "start from nothing" approach, see Suppes, *Axiomatic Set Theory*.

Notes:

Unless otherwise stated, $\mathbb{R}^n$ will be assumed to have the usual topology.

Problems:

1. (a) Consider $[0, 2\pi)$ and $\mathbb{S}^1 = \{x \in \mathbb{R}^2 \mid \|x\| = 1\}$ as subsets of $\mathbb{R}$ and $\mathbb{R}^2$, respectively, and equip these sets with the relative topology. Define a map

$$\begin{aligned} f\colon [0, 2\pi) &\to \mathbb{S}^1 \\ x &\mapsto (\cos x, \sin x) \end{aligned}$$

Is f a homeomorphism? Why or why not?

   (b) Let S be a set. Is the identity map from S with the discrete topology to S with the trivial topology continuous? a homeomorphism?

2. Show that a continuous bijection from a compact space to a Hausdorff space is always a homeomorphism. [*Hint:* Use the fact that a compact subspace of a Hausdorff space is closed.]

3. (a) Are the rational numbers a closed subset of $\mathbb{R}$? Why or why not?

   (b) The set of rational numbers, $\mathbb{Q}$, is a subset of $\mathbb{R}$ and hence inherits the usual metric on $\mathbb{R}$ to become a metric space itself. Is it a complete metric space?

4. Let $f\colon X \to \mathbb{R}$ be a continuous map and let X be compact. Show that f is bounded. That is, show that there exists $M > 0$ such that $|f(x)| \leq M$ for every $x \in X$.

5. [Abraham, Marsden, and Ratiu, Exercise 1.2C, page 12]
Show that every separable metric space is second countable.