CALIFORNIA INSTITUTE OF TECHNOLOGY
Control and Dynamical Systems

ACM/CDS 202
Problem Set #1

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Spring 2013		Due:	24 Apr 2013

Reading: Abraham, Marsden, and Ratiu, Sections 2.3–2.4, 3.1–3.3

Problems:

1. [Guillemin and Pollack, page 5, #3]
   Let M, N, and P be smooth manifolds and let $f : M \to N$ and $g : N \to P$ be smooth maps.

   (a) Show that the composite map $g \circ f : M \to P$ is smooth.

   (b) Show that if f and g are diffeomorphisms, so is $g \circ f$.

   (You may use the fact that the composition of smooth functions between open subsets of Euclidean spaces are smooth.)

2. [Boothby II.1.2] Using stereographic projection from the north pole $N(0, 0, +1)$ of all of the standard unit sphere in $\mathbb{R}^3$ except $(0, 0, +1)$ determine a coordinate neighborhood U_N, ϕ_N. In the same way determine by projection from the south pole $S(0, 0, -1)$ a neighborhood U_S, ϕ_S (see figure in Boothby). Show that these two neighborhoods determine a C^∞ structure on S^2. Generalize to S^{n-1}.

3. MTA 3.1-4 (i) and (ii): Manifold structure of the Möbius band.

   Optional: Try to use your intuition about Möbius band to answer the following questions (then try them to see if you are right):

   - Consider a a Möbius band of finite width, like the one shown in Figure 3.4.4. What happens is you cut it down the center with a pair of scissors? Is the resulting set a manifold? Is it connected?
   - Repeat the experiment, but this time cutting the Möbius band one third of the distance from one of the edges.

4. MTA 3.2-4: Submanifolds are locally closed

5. MTA 3.3-1: Graphs of manifolds