CALIFORNIA INSTITUTE OF TECHNOLOGY
Computing and Mathematical Sciences

CDS 131

R. Murray | **Homework Set #9** | Issued: 25 Nov 2020
Fall 2020 | | Due: 4 Dec 2020 (Fri)

Note: In the upper left hand corner of the *second* page of your homework set, please put the number of hours that you spent on this homework set (including reading).

1. [DFT 6.1] Show that any stable transfer function can be *uniquely* factored as the product of an all pass function and a minimum phase function, up to a choice of sign.

2. [DFT 6.5] Define $\epsilon = \|W_1 S\|_\infty$ and $\delta = \|CS\|_\infty$, so that ϵ is a measure of tracking performance and δ measures control effort. Show that for every point s_0 with Re $s_0 \geq 0$,

$$|W_1(s_0)| \leq \epsilon + |W_1(s_0)P(s_0)|\,\delta.$$

   Hence ϵ and δ cannot both be very small and so we cannot get good tracking without exerting some control effort.

For the next three problems, $M_\mathrm{s} := \|S\|_\infty$ is the maximum sensitivity, $M_\mathrm{t} := \|T\|_\infty$ is the maximum complementary sensitivity, and $s_\mathrm{m} = 1/M_\mathrm{s}$ is the stability margin.

3. [FBS 14.3] Consider a closed loop system consisting of a first-order process and a proportional controller. Let the loop transfer function be

$$L(s) = P(s)C(s) = \frac{k}{s+1},$$

   where parameter $k > 0$ is the controller gain. Show that the magnitude of the sensitivity function is bounded above by 1 and can be made arbitrarily small up to any frequency ω.

4. [FBS 14.4] In Theorem 14.1 it was assumed that $sL(s)$ goes to zero as $s \to \infty$. Assume instead that $\lim sL(s) = a$ and show that

$$\int_0^\infty \log|S(i\omega)|\,d\omega = \int_0^\infty \log\frac{1}{|1 + L(i\omega)|}\,d\omega = \pi \sum p_k - a\,\frac{\pi}{2},$$

   where p_k are the poles of the loop transfer function $L(s)$ in the right half-plane.

5. [FBS 14.14] Consider a process $P(s)$ with the right half-plane zeros z_k and right half-plane poles p_k. Introduce the polynomial $n(s)$ with zeros $s = z_k$ and the polynomial $d(s)$ with zeros $s = p_k$. Show that the complementary sensitivity function has the property

$$M_\mathrm{t} \geq \max_k \left|\frac{n(-p_k)}{n(p_k)}\right|.$$