CALIFORNIA INSTITUTE OF TECHNOLOGY
Computing and Mathematical Sciences

CDS 131

R. Murray | **Homework Set #8** | Issued: 18 Nov 2020
Fall 2020 | | Due: 25 Nov 2020

Note: In the upper left hand corner of the *second* page of your homework set, please put the number of hours that you spent on this homework set (including reading).

1. [DFT 4.6] Consider the unity feedback system with $C(s) = 10$ and plant

$$P(s) = \frac{1}{s-a},$$

where a is *real*.

(a) Find the range of a for the system to be internally stable.

(b) For $a = 0$ the plant is $P(s) = 1/s$. Regarding a as a perturbation, we can write the plant as

$$\widetilde{P} = \frac{P}{1 + \Delta W_2 P}$$

with $W_2(s) = -a$. Then $\widetilde{P}$ equals the true plant when $\Delta(s) = 1$. Apply robust stability theory to see when the feedback system $\widetilde{P}$ is internally stable for all $\|\Delta\|_\infty \leq 1$. Compare this to your result for part (a).

2. [DFT 4.10] Suppose that the plant transfer function is [10]

$$\tilde{P}(s) = [1 + \Delta(s)W_2(s)]P(s),$$

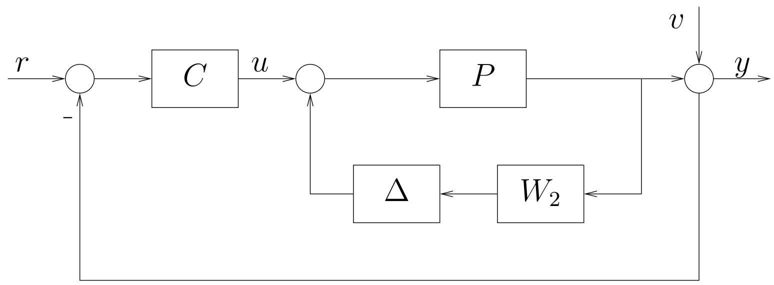
where

$$W_2(s) = \frac{2}{s+10}, \qquad P(s) = \frac{1}{s-1},$$

and the stable perturbation Δ satisfies $\|\Delta\|_\infty \leq 2$. Suppose that the controller is the pure gain $C(s) = k$. We want the feedback system to be internally stable for all such perturbations. Determine over what range of k this is true.

3. [FBS 13.2] Consider systems with the transfer functions $P_1 = (s+1)/(s+1)^2$ and $P_2 = (s+a)/(s+1)^2$. Show that P_1 can be changed continuously to P_2 with bounded feedback uncertainty if $a > 0$ but not if $a < 0$. Also show that no restriction on a is required for additive and multiplicative uncertainties.

4. Consider the system shown below. The performance objective is $\|W_1 H_{uv}\|_\infty < 1$ for all $\|\Delta\|_\infty \leq 1$, where H_{uv} is the transfer function from v to u.



(a) Derive a set of necessary and sufficient conditions for robust stability of the system.

(b) Show that a sufficient condition for robust performance is that the system is robustly stable and that the following additional condition is satisfied:

$$\left\| \frac{W_1 L(1+|W_2 P|)}{P(|1+L|-|PW_2|)} \right\|_\infty < 1.$$