CALIFORNIA INSTITUTE OF TECHNOLOGY
Computing and Mathematical Sciences

CDS 131

R. Murray	**Homework Set #6**	Issued: 4 Nov 2020
Fall 2020		Due: 11 Nov 2020

Note: In the upper left hand corner of the *second* page of your homework set, please put the number of hours that you spent on this homework set (including reading).

1. [Dullerud and Paganini, 2.19] Consider a transfer function $G(s)$ and let (A, B, C, D) be a realization: $G(s) = C(sI - A)^{-1}B + D$.

   Prove that if (A, B, C, D) is a non-minimal realization, then every eigenvalue of A is either a pole of $G(s)$ or corresponds to an eigenvector that is in the unreachable or unobservable subspace.

2. Suppose that $u(t)$ is a continuous signal whose derivative $\dot{u}(t)$ is also continuous. Which of the following quantities qualifies as a norm for u:

   (a) $\sup_t |\dot{u}(t)|$

   (b) $|u(0)| + \sup_t |\dot{u}(t)|$

   (c) $\max\{\sup_t |u(t)|, \sup_t |\dot{u}(t)|\}$

   Make sure to give a thorough answer (not just yes or no).

3. Let D be a pure time delay of τ seconds with transfer function

$$\widehat{D}(s) = e^{-s\tau}.$$

   A norm $\|\cdot\|$ on transfer functions is *time-delay invariant* if for every bounded transfer function $\widehat{G}$ and every $\tau > 0$ we have

$$\|\widehat{D}\widehat{G}\| = \|\widehat{G}\|$$

   Determine if the 2-norm and ∞-norm are time-delay invariant.

4. Derive the ∞-norm to ∞-norm system gain for a stable, proper plant $\widehat{G}$. (Hint: write $\widehat{G} = c + \widehat{G}_1$ where c is a constant and $\widehat{G}_1$ is strictly proper.)

5. Consider a system with transfer function

$$\widehat{G}(s) = \frac{s+2}{4s+1}$$

   and input u and output y. Compute the system norm given by

$$\|G\|_1 = \sup_{\|u\|_\infty = 1} \|y\|_\infty$$

   and find an input that achieves the supremum.