

CALIFORNIA INSTITUTE OF TECHNOLOGY
Computing and Mathematical Sciences

CDS 131

R. Murray
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Homework Set #5

Issued: 28 Oct 2020
Due: 4 Nov 2020

Note: In the upper left hand corner of the *second* page of your homework set, please put the number of hours that you spent on this homework set (including reading).

1. [FBS 8.2] Consider a system under a coordinate transformation $z = Tx$, where $T \in \mathbb{R}^{n \times n}$ is an invertible matrix. Show that the observability matrix for the transformed system is given by $\widetilde{W}_o = W_o T^{-1}$ and hence observability is independent of the choice of coordinates.
2. Show that the set of unobservable states for a linear system with dynamics matrix A and output matrix C is an A -invariant subspace and that it is equal to the largest A -invariant subspace annihilated by C .
3. [FBS 8.15] Consider a linear system characterized by the matrices

$$A = \begin{pmatrix} -2 & 1 & -1 & 2 \\ 1 & -3 & 0 & 2 \\ 1 & 1 & -4 & 2 \\ 0 & 1 & -1 & -1 \end{pmatrix}, \quad B = \begin{pmatrix} 2 \\ 2 \\ 2 \\ 1 \end{pmatrix}, \quad C = \begin{pmatrix} 0 & 1 & -1 & 0 \end{pmatrix}, \quad D = 0.$$

Construct a Kalman decomposition for the system. (Hint: Try to diagonalize.)

4. Consider a control system having state space dynamics

$$\frac{dx}{dt} = \begin{bmatrix} -\alpha - \beta & 1 \\ -\alpha\beta & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ k \end{bmatrix} u, \quad y = \begin{bmatrix} 1 & 0 \end{bmatrix} x.$$

- (a) Construct an observer for the system and find expressions for the observer gain $L = \begin{pmatrix} l_1 & l_2 \end{pmatrix}^T$ such that the observer has natural frequency ω_0 and damping ratio ζ .
- (b) Suppose that we choose a different output

$$\tilde{y} = \begin{bmatrix} 1 & \gamma \end{bmatrix} x.$$

Are there any values of γ for which the system is *not* observable? If so, provide an example of an initial condition and output where it is not possible to uniquely determine the state of the system by observing its inputs and outputs.

5. Show that the design of an observer by eigenvalue assignment is unique for single-output systems. Construct examples that show that the problem is not necessarily unique for systems with many outputs. (Hint: use appropriate canonical forms.)