CALIFORNIA INSTITUTE OF TECHNOLOGY
Computing and Mathematical Sciences

CDS 131

R. Murray	**Homework Set #9**	Issued: 30 Nov 2018
Fall 2018		Due: 7 Dec 2018

Note: In the upper left hand corner of the *second* page of your homework set, please put the number of hours that you spent on this homework set (including reading).

1. [DFT 6.4] Let
$$P(s) = 4\frac{s-2}{(s+1)^2}$$
and suppose that C is an internally stabilizing controller such that $\|S\|_\infty = 1.5$. Give a positive lower bound for
$$\max_{0\le\omega\le 0.1} |S(j\omega)|.$$

2. [DFT 6.5] Define $\epsilon = \|W_1 S\|_\infty$ and $\delta = \|CS\|_\infty$, so that ϵ is a measure of tracking performance and δ measures control effort. Show that for every point s_0 with Re $s_0 \ge 0$,
$$|W_1(s_0)| \le \epsilon + |W_1(s_0)P(s_0)|\,\delta.$$
Hence ϵ and δ cannot both be very small and so we cannot get good tracking without exerting some control effort.

3. [DFT 6.7] Consider a plant with transfer function
$$P(s) = \frac{1}{s^2 - s + 4}$$
and suppose we want to design an internally stabilizing controller such that
   (a) $|S(j\omega| \le \epsilon$ for $0 \le \omega \le 0.1$
   (b) $|S(j\omega| \le 2$ for $0.1 \le \omega \le 5$
   (c) $|S(j\omega| \le 1$ for $5 \le \omega \le \infty$

   Find a (positive) lower bound on the achievable ϵ.

4. [FBS 14.4] In Theorem 14.1 it was assumed that $sL(s)$ goes to zero as $s \to \infty$. Assume instead that $\lim sL(s) = a$ and show that
$$\int_0^\infty \log|S(i\omega)|\,d\omega = \int_0^\infty \log\frac{1}{|1+L(i\omega)|}\,d\omega = \pi\sum p_k - a\,\frac{\pi}{2},$$
where p_k are the poles of the loop transfer function $L(s)$ in the right half-plane.

5. [FBS 14.14] Consider a process $P(s)$ with the right half-plane zeros z_k and right half-plane poles p_k. Introduce the polynomial $n(s)$ with zeros $s = z_k$ and the polynomial $d(s)$ with zeros $s = p_k$. Show that the complementary sensitivity function has the property

$$M_{\mathrm{t}} \geq \max_k \left| \frac{n(-p_k)}{n(p_k)} \right|.$$

Also show that the equations (14.29) hold.

6. [FBS 14.16] Consider a process with the transfer function

$$P(s) = \frac{e^{-s\tau}}{s - p} \bar{P}(s),$$

where $\bar{P}(s)$ has no poles and zeros in the right half-plane. Show that the sensitivity functions have the properties listed in Table 14.1:

$$M_{\mathrm{t}} \geq e^{p\tau}, \qquad M_{\mathrm{s}} \geq e^{p\tau} - 1.$$