CALIFORNIA INSTITUTE OF TECHNOLOGY
Computing and Mathematical Sciences

CDS 131

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Fall 2018

Homework Set #8

Issued: 21 Nov 2018
Due: 30 Nov 2018

Note: In the upper left hand corner of the *second* page of your homework set, please put the number of hours that you spent on this homework set (including reading).

1. [DFT 4.4] Suppose that
$$P(s) = \frac{\omega_n^2}{s(s + 2\zeta\omega_n)} \qquad C(s) = 1$$
with $\omega_n, \zeta > 0$. Plot the phase margin as a function of ζ.

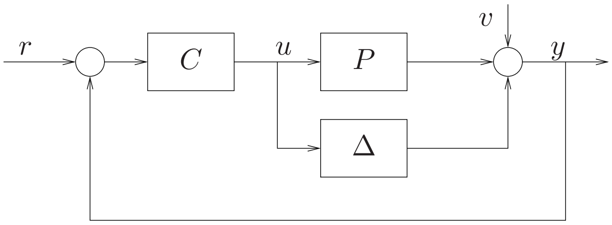
2. [DFT 4.6] Consider the unity feedback system with $C(s) = 10$ and plant
$$P(s) = \frac{1}{s - a},$$
where a is *real*.
   (a) Find the range of a for the system to be internally stable.
   (b) For $a = 0$ the plant is $P(s) = 1/s$. Regarding a as a perturbations, we can write the plant as
$$\widetilde{P} = \frac{P}{1 + \Delta W_2 P}$$
with $W_2(s) = -a$. Then $\widetilde{P}$ equals the true plant when $\Delta(s) = 1$. Apply robust stability theory to see when the feedback system $\widetilde{P}$ is internally stable for all $\|\Delta\|_\infty \leq 1$. Compare this to your result for part (a).

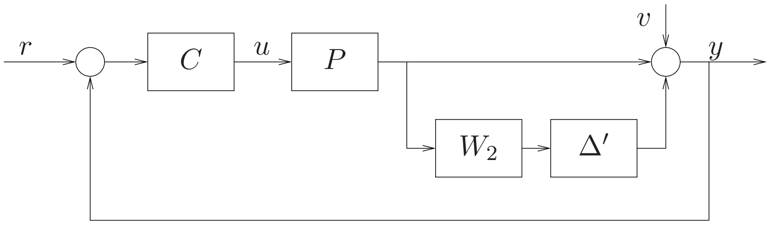
3. [DFT 4.9] Consider the class of perturbed plants of the form
$$\frac{P}{1 + \Delta W_2 P},$$
where W_2 is a fixed stable weighting function with $W_2 P$ strictly proper and Δ is a variable, stable transfer function with $\|\Delta\|_\infty \leq 1$. Assume that C is a controller achieving internal stability for the nominal plant P. Prove that C provides internal stability for the perturbed plant if $\|W_2 P S\|_\infty < 1$.

4. [FBS 13.1] Consider systems with the transfer functions $P_1 = 1/(s + 1)$ and $P_2 = 1/(s + a)$. Show that P_1 can be changed continuously to P_2 with bounded additive and multiplicative uncertainty if $a > 0$ but not if $a < 0$. Also show that no restriction on a is required for feedback uncertainty.

5. In this problem we will transform an additive uncertainty problem to a multiplicative uncertainty problem in order to get a closed form solution for the robust performance problem.

Consider the two unity feedback loops shown below with the uncertainty in the first system given by Δ stable, $\|\Delta\|_\infty \leq 1$ and the P stable, minimum phase, and biproper.





(a) Find a stable W_2 and stable Δ' with $\|\Delta'\|_\infty \leq 1$ such that the second system is the same as the first.

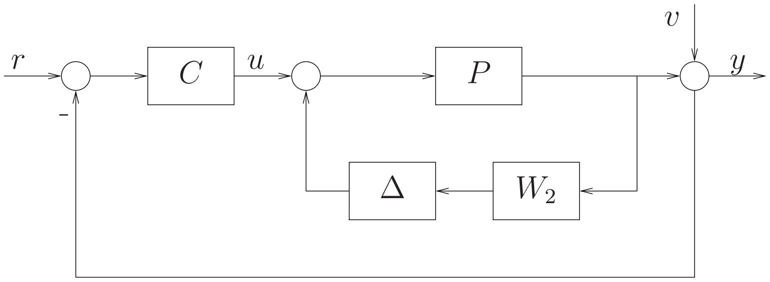
(b) Consider the performance specification given by $\|H_{uv}\|_\infty < 1$ for all Δ, where H_{uv} is the transfer function from v to u. Derive a necessary and sufficient condition for robust performance in terms of the complementary sensitivity function for the nominal plant and the weight W_2.

(c) Which of the following conditions is necessary in order for the above procedure to work:

- (i) P stable
- (ii) P minimum phase
- (iii) P biproper

Explain your answer.

6. Consider the system shown below. The performance objective is $\|W_1 H_{uv}\|_\infty < 1$ for all $\|\Delta\|_\infty < 1$, where H_{uv} is the transfer function from v to u.



(a) Derive a set of necessary and sufficient conditions for robust stability of the system.

(b) Derive a set of necessary and sufficient conditions for robust performance. These conditions may be written in terms of W_1, W_2, L and P, but should not contain C or Δ.