CALIFORNIA INSTITUTE OF TECHNOLOGY
Computing and Mathematical Sciences

CDS 131

R. Murray | **Homework Set #7** | Issued: 14 Nov 2018
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Note: In the upper left hand corner of the *second* page of your homework set, please put the number of hours that you spent on this homework set (including reading).

1. [DFT 3.1] Show that for a unity feedback system it suffices to check only two transfer functions to determine internal stability.

2. [DFT 3.2] Let
$$\widehat{P}(s) = \frac{1}{10s+1} \qquad \widehat{C}(s) = k \qquad \widehat{F}(s) = 1.$$
Find the least positive gain k such that the following are all true:
   (a) The feedback system is internally stable
   (b) $|e(\infty)| \leq 0.1$ when $r(t)$ is the unit step and $n = d = 0$.
   (c) $\|y\|_\infty \leq 0.1$ for all $d(t)$ such that $\|d\|_2 \leq 1$ when $r = n = 0$.

3. [DFT 3.3] Consider a unity gain feedback system with $r = n = 0$ and $d(t) = \sin(\omega(t)1(t)$. Prove that if the feedback system is internally stable then $y(t) \to 0$ as $t \to \infty$ if and only if $\widehat{P}$ has a zero at $s = j\omega$ or $\widehat{C}$ has a pole at $s = j\omega$.

4. Consider a linear input/output system Σ with a minimal realization given by (A, B, C, D) and let the associated transfer function be $H(s) = C(sI - A)^{-1}B + D$. For simplicity, you may also assume that the system is SISO.
   (a) Show that if the linear system $\dot{x} = Ax$ is asymptotically stable then the induced input/output norm of the system Σ is bounded.
   (b) Show that if a linear input/output system Σ has bounded induced input/output norm, then the linear system $\dot{x} = Ax$ is asymptotically stable.
   (c) Show that if a linear system is input/output stable then $\|H\|_\infty$ is bounded.
   (d) Show via example that $\|H\|_\infty$ being bounded is not a sufficient condition for stability of the underlying system.

5. [FBS 12.2] Consider the system in Figure 12.1 and let the outputs of interest be $\xi = (\mu, \eta)$ and the major disturbances be $\chi = (w, v)$. Show that the system can be represented by Figure 12.2 and give the matrix transfer functions $\mathcal{P}$ and $\mathcal{C}$. Verify that the elements of the closed loop transfer function $H_{\xi\chi}$ are the Gang of Four.

6. [FBS 12.8] Consider the feedback system shown in Figure 12.1. Assume that the reference signal is constant. Let y_{ol} be the measured output when there is no feedback and y_{cl} be the output with feedback. Show that $Y_{\text{cl}}(s) = S(s)Y_{\text{ol}}(s)$, where Y_{cl} and Y_{ol} are exponential signals and S is the sensitivity function.

7. [FBS 13.6] Bode's ideal loop transfer function is given in Example 13.8. Show that the phase margin is φ_{m} =180°–90°n and that the stability margin is $s_{\mathrm{m}} = \arcsin \pi(1 - n/2)$. Make Bode and Nyquist plots of the transfer function for n=5/3.