CALIFORNIA INSTITUTE OF TECHNOLOGY
Computing and Mathematical Sciences

CDS 131

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Note: In the upper left hand corner of the *second* page of your homework set, please put the number of hours that you spent on this homework set (including reading).

1. [FBS 8.1] Consider the system given by

$$\frac{dx}{dt} = Ax + Bu, \qquad y = Cx,$$

where $x \in \mathbb{R}^n$, $u \in \mathbb{R}^p$, and $y \in \mathbb{R}^q$. Show that the states can be determined from the input u and the output y and their derivatives if the observability matrix W_o given by equation (8.4) has n independent rows.

2. [FBS 8.2] Consider a system under a coordinate transformation $z = Tx$, where $T \in \mathbb{R}^{n\times n}$ is an invertible matrix. Show that the observability matrix for the transformed system is given by $\widetilde{W}_o = W_o T^{-1}$ and hence observability is independent of the choice of coordinates.

3. [FBS 8.4] Show that if a system is observable, then there exists a change of coordinates $z = Tx$ that puts the transformed system into observable canonical form.

4. [FBS 8.9] Consider the linear system (8.2), and assume that the observability matrix W_o is invertible. Show that

$$\hat{x} = W_o^{-1} \begin{pmatrix} y & \dot{y} & \ddot{y} & \cdots & y^{(n-1)} \end{pmatrix}^T$$

is an observer. Show that it has the advantage of giving the state instantaneously but that it also has some severe practical drawbacks.

5. [FBS 8.15] Consider a linear system characterized by the matrices

$$A = \begin{pmatrix} -2 & 1 & -1 & 2 \\ 1 & -3 & 0 & 2 \\ 1 & 1 & -4 & 2 \\ 0 & 1 & -1 & -1 \end{pmatrix}, \quad B = \begin{pmatrix} 2 \\ 2 \\ 2 \\ 1 \end{pmatrix}, \quad C = \begin{pmatrix} 0 & 1 & -1 & 0 \end{pmatrix}, \quad D = 0.$$

Construct a Kalman decomposition for the system. (Hint: Try to diagonalize.)

6. Consider the system

$$\frac{dx}{dt} = \begin{pmatrix} -4 & 1 \\ -6 & 1 \end{pmatrix} x + \begin{pmatrix} 3 \\ 7 \end{pmatrix} u, \qquad y = \begin{pmatrix} 1 & -1 \end{pmatrix} x.$$

Transform the system to observable canonical form.

7. Consider a control system having state space dynamics

$$\frac{dx}{dt} = \begin{bmatrix} -\alpha-\beta & 1 \\ -\alpha\beta & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ k \end{bmatrix} u, \qquad y = \begin{bmatrix} 1 & 0 \end{bmatrix} x.$$

(a) Construct an observer for the system and find expressions for the observer gain $L = \begin{pmatrix} l_1 & l_2 \end{pmatrix}^T$ such that the observer has natural frequency ω_0 and damping ratio ζ.

(b) Suppose that we choose a different output

$$\tilde{y} = \begin{bmatrix} 1 & \gamma \end{bmatrix} x.$$

Are there any values of γ for which the system is *not* observable? If so, provide an example of an initial condition and output where it is not possible to uniquely determine the state of the system by observing its inputs and outputs.