CALIFORNIA INSTITUTE OF TECHNOLOGY
Computing and Mathematical Sciences

CDS 112/Ae 103b

R. Murray | **Homework Set #7** | Issued: 15 Feb 2023
Winter 2023 | | Due: 22 Feb 2023

Problem 1. Suppose that we wish to estimate the position of a particle that is undergoing a (continuous) random walk in one dimension (i.e., along a line). We model the position of the particle as

$$\dot{x} = u,$$

where x is the position of the particle and u is a white noise processes with $\mathbb{E}(u(t)) = 0$ and $\mathbb{E}(u(t_1)\, u(t_2)) = \sigma^2 \delta(t_2 - t_1)$. We assume that we can measure x subject to additive, zero-mean, Gaussian white noise with covariance 1.

(a) Compute the expected value and covariance of the particle as a function of t.

(b) Construct a Kalman filter to estimate the position of the particle given the noisy measurements of its position. Compute the steady-state expected value and covariance of the error of your estimate.

(c) Suppose that $\mathbb{E}(u) = \mu \neq 0$ but is otherwise unchanged. How would your answers to parts (a) and (b) change?

Problem 2. [based on Friedland 11.1] A compensator based on a Kalman filter is to be designed for an instrument servo whose dynamics are given by

$$\dot{\theta} = \omega, \qquad \dot{\omega} = -\alpha\omega + \beta u + V,$$

where V is white noise of spectral density R_V. Only the position θ is measured, so that

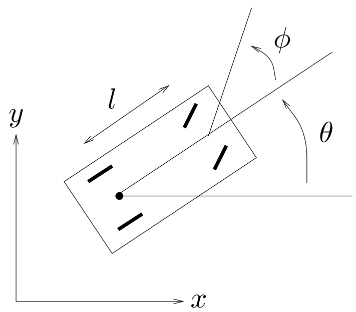
$$Y = \theta + W,$$

where W is white noise with spectral density R_W. Use parameters $\alpha = 1$ and $\beta = 3$.

(a) Show that the steady state Kalman filter gains depend only on the "signal-to-noise ratio" (SNR) given by R_V/R_W. (You do not have to solve explicitly for the gains.)

(b) Find and plot the steady state Kalman filter gains and the corresponding closed-loop poles of the estimator as a function of the signal-to-noise ratio ranging from 0.01 to 100.

Problem 3. Consider the problem of estimating the position of an autonomous mobile vehicle using a GPS receiver and an IMU (inertial measurement unit). The dynamics of the vehicle are given by



$$
\begin{aligned}
\dot{x} &= \cos\theta\, v \\
\dot{y} &= \sin\theta\, v \\
\dot{\theta} &= \frac{1}{\ell}\tan\delta\, v,
\end{aligned}
$$

We assume that the vehicle has disturbances in the inputs v and δ with standard deviation of up to 10% and noisy measurements from the GPS receiver and IMU.

We consider a trajectory in which the car is driving on a constant radius curve at $v = 10$ m/s forward speed with $\delta = 5^\circ$ for a duration of 10 seconds.

(a) Suppose first that we only have the GPS measurements for the xy position of the vehicle. These measurements give the position of the vehicle with approximately 10 cm accuracy. Model the GPS error as Gaussian white noise with $\sigma = 0.1$ meter in each direction. Design a Kalman filter-based estimator for the system and plot the estimated states versus the actual states. What is the covariance of the estimate at the end of the trajectory?

(b) An IMU can be used to measure angular rates and linear acceleration. For simplicity, we assume that the IMU is able to directly measure the angle of the car with a standard deviation of 1 degree. Design an updated estimator for the system using the GPS and IMU measurements, and plot the estimated states versus the actual states. What is the covariance of the estimate at the end of the trajectory?