CALIFORNIA INSTITUTE OF TECHNOLOGY
Computing and Mathematical Sciences

CDS 112/Ae 103a

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Winter 2022		Due: 19 Jan 2022

Problem 1. Consider the trajectory generation problem for the system

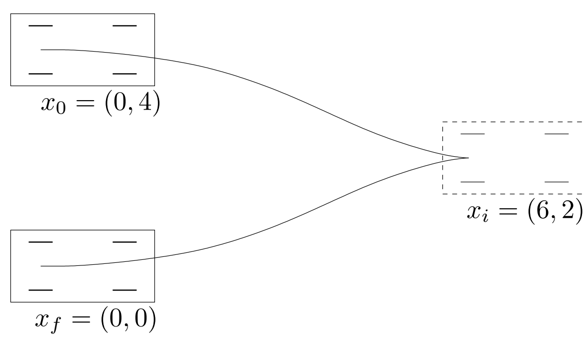
$$\frac{dx}{dt} = -ax^3 + bu,$$

where $x \in \mathbb{R}$ is a scalar state, $u \in \mathbb{R}$ is the input, the initial state $x(t_0)$ is given, and $a, b \in \mathbb{R}$ are positive constants.

(a) Show that the system is differentially flat with appropriate choice of output(s) and compute the state and input as a function of the flat output(s).

(b) Using the polynomial basis $\{t^k,\ k = 0, \ldots, M\}$ with an appropriate choice of M, solve for the (non-optimal) trajectory between $x(t_0)$ and $x(t_\mathrm{f})$. Your answer should specify the explicit input $u_\mathrm{d}(t)$ and state $x_\mathrm{d}(t)$ in terms of t_0, t_f, $x(t_0)$, $x(t_\mathrm{f})$, and t.

Problem 2. Consider the lateral control problem for an autonomous ground vehicle as described in Example 2.1 and Section 2.3 in OBC. Using the fact that the system is differentially flat, find an explicit trajectory that solves the following parallel parking maneuver:



Your solution should consist of two segments: a curve from x_0 to x_i with $v > 0$ and a curve from x_i to x_f with $v < 0$. For the trajectory that you determine, plot the trajectory in the plane (x versus y) and also the inputs v and ϕ as a function of time.

Problem 3. A simplified model of the steering control problem is described in FBS2e, Example 6.13. The lateral dynamics can be approximated by the linearized dynamics

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} x + \begin{bmatrix} \gamma \\ 1 \end{bmatrix} u, \qquad y \ = x_1,$$

where $x = (y, \theta) \in \mathbb{R}^2$ is the state of the system, γ is a parameter that depends on the forward speed of the vehicle, and y is the lateral position of the vehicle.

Suppose that we wish to track a piecewise constant reference trajectory that consists of moving left and right by 1 meter:

$$x_{\mathrm{d}} = \begin{bmatrix} \texttt{square}(2\pi t/20) \\ 0 \end{bmatrix}, \qquad u_{\mathrm{d}} = 0,$$

where `square` is the square wave function in `scipy.signal`. Suppose further that the speed of the vehicle varies such that the parameter γ satisfies the formula

$$\gamma(t) = 2 + 2\sin(2\pi t/50).$$

(a) Show that the desired trajectory given by x_{d} and u_{d} satisfy the dynamics of the system at all points in time except the switching points of the square wave function.

(b) Suppose that we fix $\gamma = 2$. Use eigenvalue placement to design a state space controller $u = u_{\mathrm{d}} - K(x - x_{\mathrm{d}})$ where the gain matrix K is chosen such that the eigenvalues of the closed loop poles are at the roots of $s^2 + 2\zeta\omega_0 s + \omega_0^2$, where $\zeta = 0.7$ and $\omega_0 = 1$. Apply the controller to the time-varying system where $\gamma(t)$ is allowed to vary and plot the output of the system compared to the desired output.

(c) Find gain matrices K_1, K_2, and K_3 corresponding to $\gamma = 0, 2, 4$ and design a gain-scheduled controller that uses linear interpolaction to compute the gain for values of γ between these values. Compare the performance of the gain scheduled controller to your controller from part (b).