CALIFORNIA INSTITUTE OF TECHNOLOGY
Control and Dynamical Systems

CDS 110b

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Winter 2008

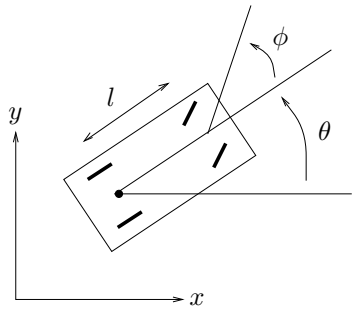
Problem Set #7

Issued: 27 Feb 08
Due: 5 Mar 08

Note: Please put the number of hours that you spent on this homework set (including reading) on the back of the first page of your homework.

Students who are not doing the course project should complete the following problem (worth 20 points):

1. Consider the problem of estimating the position of an autonomous mobile vehicle using a GPS receiver and an IMU (inertial measurement unit). The continuous time dynamics of the vehicle are given by



$$\dot{x} = \cos\theta\, v$$
$$\dot{y} = \sin\theta\, v$$
$$\dot{\theta} = \frac{1}{\ell}\tan\phi\, v,$$

We assume that the vehicle is disturbance free, but that we have noisy measurements from the GPS receiver and IMU and an initial condition error.

(a) Rewrite the equations of motion in discrete time, assuming that we update the dynamics at a sample time of $h = 0.005$ sec and that we can take $\dot{x}$ to be roughly constant over that period. Run a simulation of your discrete time model from initial condition $(0, 0, 0)$ with constant input $\phi = \pi/8$, $v = 5$ and compare your results with the continuous time model.

(b) Suppose that we have a GPS measurement that is taken every 0.1 seconds and an IMU measurement that is taken every 0.01 seconds. Write a MATLAB program that that computes the discrete time Kalman filter for this system, using the same disturbance, noise and initial conditions as Exercise 6.1.