CALIFORNIA INSTITUTE OF TECHNOLOGY
Control and Dynamical Systems

CDS 110/ChE 105 PRACTICE Exam
R. Murray, 3 June 2024

The exam is closed book. You may use a single sheet of regular paper with notes on one side as a study aid. No other materials are allowed, including the use of cell phones, tablets, or computers.

For each of the multiple choice answers below, there may be one, more than one, or zero correct answers. You should circle all that apply.

You only need to provide justification for your answers if it is specifically requested. Otherwise, just circle T, F, or the letter corresponding to the correct answer for multiple choice questions.

NOTE: This practice exam is not meant to cover all topics that will be on the exam, but to give you a sense of the different types of questions that will be on the exam. You are responsible for all material covered in HW1-HW9 (HW8 for seniors and graduate students).

Name: ____________________

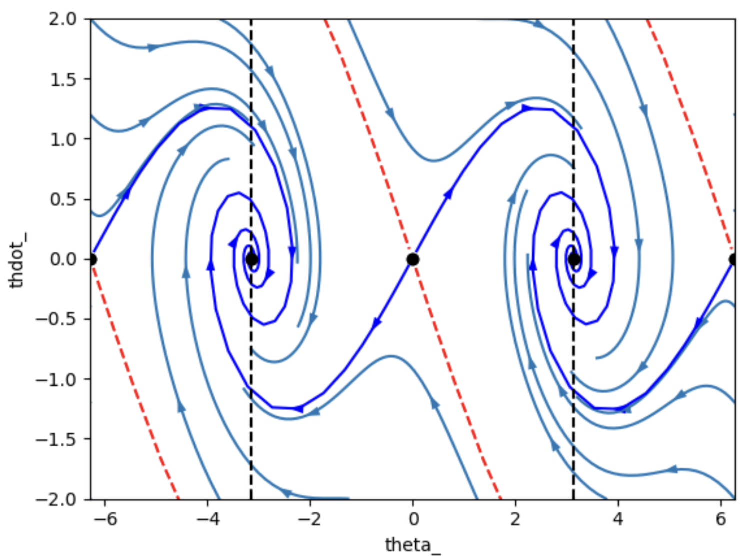
UID: ____________________

Date: ____________________

1. Which of the following python-control commands can be used to numerically compute the steady state response of a closed-loop, linear control system to a sinusoidal input?

   A. `ct.input_output_response` B. `ct.create_statefbk_system` C. `ct.step_response`
   D. `ct.frequency_response` E. `ct.bode_plot` F. `ct.gangof4`

2. Consider the following phase portrait for a nonlinear dynamical system:



(a) Which of the following initial conditions converge to a stable equilibrium point of the system?

A. (0, 0) B. (1, 0) C. (0, 1)

(b) For the equilibrium point at $(\pi, 0)$, what are the possible eigenvalues for the system:

A. (0, 0) B. (-1, -1) C. (-1, 1) D. (j, -j) E. (1 + j, 1 - j)

3. Consider the following linear system

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ -a & -b \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u, \qquad y = \begin{bmatrix} 0 & c \end{bmatrix}.$$

(a) Which of the following differential equations can be represented as a state space system in the given form?

A. $\ddot{y} + a\dot{y} + by = u$

B. $\ddot{y} + a\dot{y} + by = cu$

C. $\ddot{y} - a\dot{y} - by = cu$

D. $a\ddot{y} + b\dot{y} + cy = u$

E. $a\ddot{y} + a^2\dot{y} + aby = acu$

(b) Which of the following conditions gaurantee that the system is stable at the origin?

A. $a > 0$ B. $a, b > 0$ C. $a > 0,\, b < 0$ D. $a > 0,\, c > 0$ E. $a > 0,\, b > a$

(c) Which of the following conditions gaurantee that the system is reachable?

A. $a > 0$ B. $a, b > 0$ C. $a > 0,\, b < 0$ D. $a > 0,\, c > 0$ E. $a > 0,\, b > a$

(d) Which of the following conditions gaurantee that the system is observable?

A. $a > 0$ B. $a, b > 0$ C. $a > 0,\, b < 0$ D. $a > 0,\, c > 0$ E. $a > 0,\, b > a$

4. Suppose that we want to solve an optimal control problem of the form

$$\min \int_0^T x^{\mathtt{T}}(\tau) Q_x x(\tau) + u^{\mathtt{T}}(\tau) Q_u u(\tau)\, d\tau + (x(T) - x_{\mathrm{f}})^{\mathtt{T}} Q_x (x(T) - x_{\mathrm{f}})$$

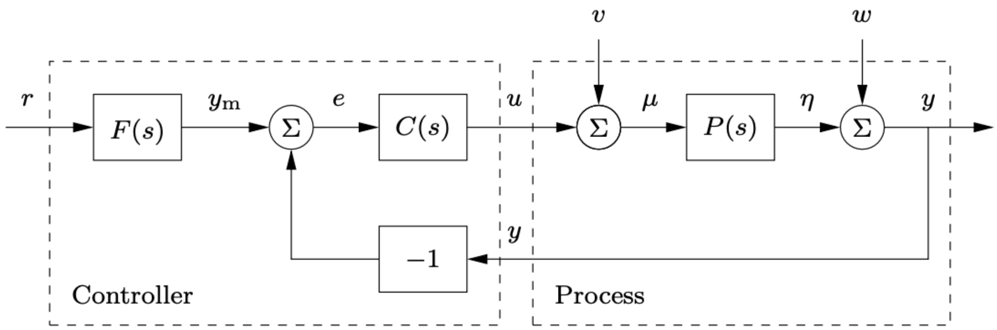
Modify the Python code below by filling in any blanks and crossing out any commands (or portions of commands) that are not needed. (You can assume that `sys`, `Qx`, `Qu`, `x0`, `xf`, and `T` are already defined.)

```
# Call signature: quadratic_cost(sys, Qx, Qu, x0=0, u0=0)
cost1 = opt.quadratic_cost(sys, ____, Qu)
cost2 = opt.quadratic_cost(sys, Qx, Qu, x0=____)
cost3 = opt.quadratic_cost(sys, Qx, ____, x0=xf)


# Call signature: state_range_constraint(sys, lb, ub)
constraint1 = opt.state_range_constraint(sys, 0, ____)
constraint2 = opt.state_range_constraint(sys, ____, xf)
```

```
timepts = np.linspace(_____, _____, 10)
soln = opt.solve_ocp(
    sys, timepts, X0=x0, cost=____________,
    terminal_cost=____________,
    trajectory_constraint=________________,
    terminal_constraint=________________,
)
```

5. Consider the following block diagram:



Which of the following statements are true?

A. The transfer function from v to u is given by $PC/(1 + PC)$.

B. The transfer function from w to e is given by $PC/(1 + PC)$.

C. If $P(s)$ has all of its poles in the left half-plane, then the number of encirclements of the -1 point by the Nyquist curve must be zero for the closed loop system to be stable.

D. The Bode integral formula limits the achievable bandwidth for a closed loop system.

E. For a system with $F(s) = 1$, the Bode integral formula implies that for some frequency of the reference signal r, the magnitude of the error $e = r - y$ will be larger that the magnitude of the reference.

F. A controller with a right half-plane zero will always create an unstable closed loop system.

G. A controller with a right half-plane zero will limit the achievable performance of a stable closed loop system.