CALIFORNIA INSTITUTE OF TECHNOLOGY
Computing and Mathematical Sciences

**CDS 110/ChE 105**

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**Problem 1.** Consider the system in Example 12.1 in FBS2e, where the process and controller transfer functions are given by

$$P(s) = 1/(s-a), \qquad C(s) = k(s-a)/s.$$

Notice the pole-zero cancellation.

(a) Let the controller gain be $k = 0.5$. For each transfer function in the Gang of Four, plot the frequency responses for $a = -1$ and $a = 1$.

(b) For the same sets of parameter values, plot the step responses corresponding to each transfer function in the Gang of Four.

(c) Compare and contrast each pair of step responses for the two systems: Which are different, and which are the same? Explain why.

**Problem 2.** Consider the balance system described in Example 3.2 of FBS2e, using the following parameters:

$$\begin{aligned} M &= 10 \text{ kg}, & m &= 80 \text{ kg}, & J &= 100 \text{ kg m}^2, \\ c &= 0.1 \text{ N/m/sec}, & l &= 1 \text{ m}, & \gamma &= 0.01 \text{ Nms}, \end{aligned} \qquad g = 9.8 \text{ m/s}^2.$$

In this problem we will design a (state feedback-based) controller for this system and explore the limits of performance for the closed loop system.

(a) Compute the linearization of the dynamics about the equilibrium point corresponding to the upright position. Find the poles and zeros and indicate whether they are "stable" (LHP) or "unstable" (RHP).

For the remainder of this problem, you should just use the linearized dynamics of the system (small signal analysis).

(b) Assume that we want to bound the sensitivity function by a transfer function of the form

$$S_\mathrm{r} = \frac{M_\mathrm{s} s}{s+a},$$

where $M_\mathrm{s} > 1$ and $a > 0$. Give a bound on the minimum value of $M_\mathrm{s}$ and, for that minimum value, give a bound on the maximum value of $a$.

(c) Design an LQR controller for the system that stabilizes the system about an equilibrium point with $x_\mathrm{d} = (r, 0, 0, 0)$, $u_\mathrm{d} = 0$. Plot the unit step response for the closed loop system, and compute the rise time, overshoot, and settling time. Choose the weights of your system so that the settling time is less than 10 seconds.

(d) Construct an optimal estimator for the system assuming noise intensities given by $R_d = 0.1$ and $R_n = 0.01$. Plot the initial condition response for the estimator from an initial position of $\hat{x}(0) = (0.1, 0.1, 0, 0)$.

(e) Design an estimation-based controller for the system that stabilizes the system and tracks a reference command for the center of mass of the rider. Plot the unit step response for the closed loop system and compute the rise time, overshoot, and settling time. Comment on any differences in the response versus the state feedback controller.

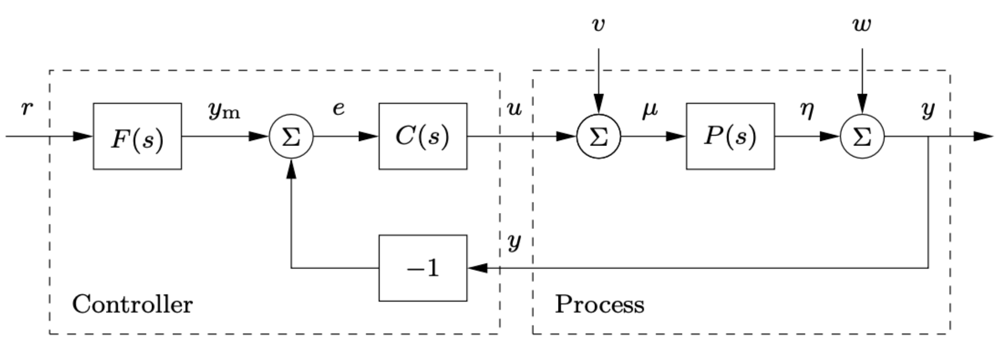
(f) Compute the sensitivity function for the closed loop system and plot its frequency response. Verify that the maximum sensitivity for your design is above the bound computed in (b).

(g) Compute the Bode integral for sensitivity of the closed loop system and verify that it matches the expected value.

A Colab notebook template for this problem is available here.

*The following problem is a conceptual problem that you should be able to do without using the textbook, your notes, or the Internet. It is representative of the type of problem you can expect on the (closed-book) final exam (so you might want to first attempt solving it without using any notes).*

**Problem 3.** Answer the following questions regarding frequency domain analysis. Assume that we are considering the standard small signal control system given in the following diagram:




For each question, you should provide the answer and a *short* (1-2 sentence max) justification.

(a) T/F: The Bode integral formula limits the achievable bandwidth for a closed loop system.

(b) T/F: The Bode integral formula implies that for some frequency of the reference signal $r$, the magnitude of the error $e = r - y$ will be larger that the magnitude of the reference.

(c) T/F: A controller with a right half-plane zero will always create an unstable closed loop system.

(d) T/F: A controller with a right half-plane zero will limit the achievable performance of a stable closed loop system.

(e) T/F: If the magnitude of the loop transfer function is greater than 1 at a frequency where the phase of the loop transfer function is 180°, then the closed loop system will always be unstable.

(f) T/F: Assuming $L(s) = 0$ as $s \to \infty$, if the loop transfer function has not poles in the right half-plane and the magnitude of the loop transfer function is less than 1 at any frequency where the phase of the loop transfer function is $180^\circ$, then the closed loop system will never be unstable.