CALIFORNIA INSTITUTE OF TECHNOLOGY
Computing and Mathematical Sciences

CDS 110/ChE 105

R. Murray | **Problem Set #7** | Issued: 15 May 2024
Spring 2024 | | Due: 22 May 2024

Problem 1.

(a) Consider the first-order system

$$\frac{dx}{dt} = ax + u.$$

Compute the exponential response of the system and use this to derive the transfer function from u to x.

(b) Show that when $s = a$, a pole of the first-order transfer function, the response to the exponential input $u(t) = e^{st}$ is $x(t) = e^{at}x(0) + te^{at}$.

(c) Consider the second-order system

$$A = \begin{pmatrix} 0 & 1 \\ \alpha & 0 \end{pmatrix}, \quad B = \begin{pmatrix} 0 \\ \beta \end{pmatrix}, \quad C = \begin{pmatrix} 1 & 0 \end{pmatrix}, \quad D = 0,$$

where $\alpha > 0$ and $\beta > 0$. Determine the transfer function of the system.

(d) Compute the poles and zeros of the second-order transfer function and determine whether the system is stable, asymptotically stable, unstable.

Problem 2. Consider the speed control system described in Section 4.1 of FBS2e. We wish to model the dynamics of the system on on flat ground ($\theta = 0$), in fourth gear, and with desired speed near 20 m/s.

We will create a feedback controller for the system using the modified PI controller

$$C(s) = G_{ue}(s) = k_\text{p} + \frac{k_\text{i}}{s+\beta} = \frac{k_\text{p}s + k_\text{i} + k_\text{p}\beta}{s+\beta} \tag{S7.1}$$

where $k_\text{p} = 0.5$, $k_\text{i} = 0.1$, and $\beta = 0.1$.

We also assume that there is a speed sensor for the system that has a short time delay, which we model with the transfer function

$$G(s) = \frac{1 - \tau s/2 + (\tau s)^2/12}{1 + \tau s/2 + (\tau s)^2/12}.$$

It can be shown these dynamics provide an approximation to a pure time delay of length τ, and the numerator and denominator polynomial coefficient arrays can be computed using the command `num, den = ct.pade(tau, 2)`.

(a) Implement the control system as an linear state space system and show that the transfer function matches the transfer function (S7.1).

(b) Plot the (open loop) frequency responses of the vehicle dynamics, sensor dynamics, control system, and loop transfer function. Use a single plot, with each curve in a different color and a legend to identify the different components.

(c) Plot the Nyquist curve for the system and use the Nyquist plot to estimate the gain, phase, and stability margins.

(d) Plot the frequency responses for the closed loop system between the reference input r and the system output y. Determine the bandwidth the closed loop system.

(e) Plot the response of the closed loop system to a step change in r of 10 m/s (corresponding to increasing the speed from the nominal speed ~40 mph to ~60 mph). Determine the settling time and overshoot of the response. (Note: you will see some unusual dynamics at the start of your step response due to the use of the Pade approximation instead of a pure delay.)

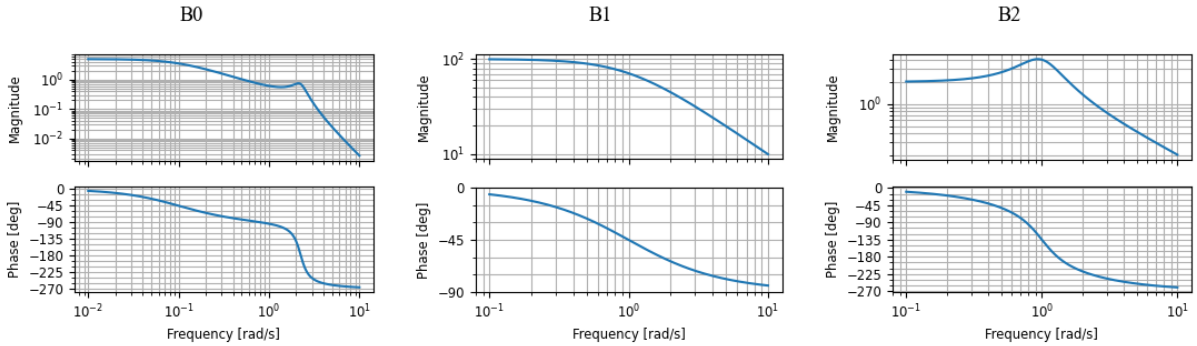
A template containing the dynamics of the system is available here.

The following problem is a conceptual problem that you should be able to do without using the textbook, your notes, or the Internet. It is representative of the type of problem you can expect on the (closed-book) final exam (so you might want to first attempt solving it without using any notes).

Problem 3. In this problem, you will match up frequency responses, Nyquist plots, and step responses with one of the loop transfer functions:

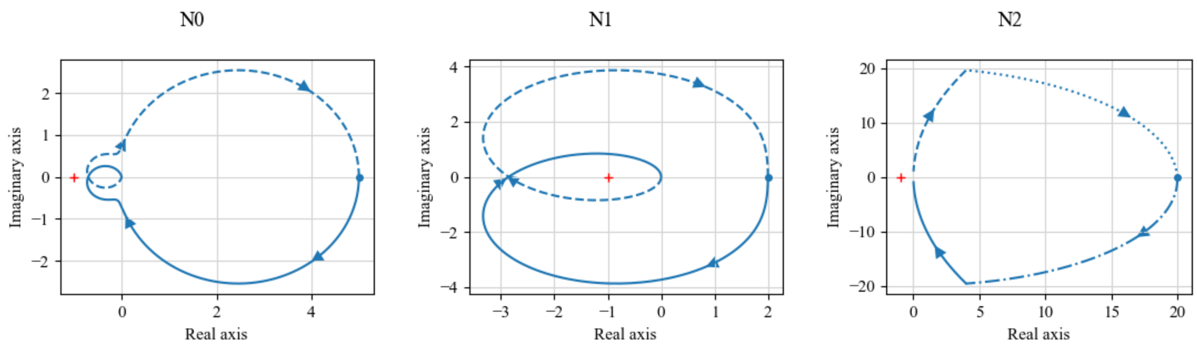
$$L_1(s) = \frac{k}{s+a}, \qquad L_2(s) = \frac{k(-s+a)}{s^2 + 2\zeta\omega_0 s + \omega_0^2}, \qquad L_3(s) = \frac{k}{s+a} \cdot \frac{\omega_0^2}{s^2 + 2\zeta\omega_0 s + \omega_0^2}.$$

(a) For each of the Bode plots below, indicate which of the systems it corresponds to:



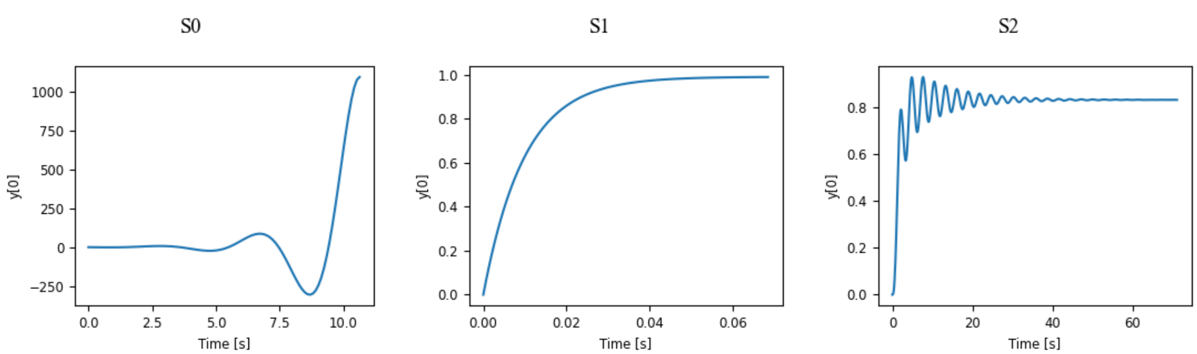
Provide a brief justification for your answers.

(b) For each of the Nyquist plots below, indicate which of the systems it corresponds to:



Provide a brief justification for your answers.

(c) For each of the *closed loop* step responses below (for $H_{yr} = L/(1+L)$), indicate which of the systems it corresponds to:



Provide a brief justification for your answers.