CALIFORNIA INSTITUTE OF TECHNOLOGY
Computing and Mathematical Sciences

CDS 110/ChE 105

R. Murray | **Problem Set #5** | Issued: 2 May 2024
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Problem 1. Consider a normalized inverted pendulum with a rate sensor described by

$$\frac{d}{dt}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} u = Ax + Bu, \qquad y = \begin{pmatrix} 0 & 1 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = Cx.$$

(a) Is this system reachable? Is it observable?

(b) Design a controller based on state feedback and an observer such that the matrices $A - BK$ and $A - LC$ have the characteristic polynomials $s^2 + a_1 s + a_2$ and $s^2 + b_1 s + b_2$ with all coefficients positive.

(c) Construct a state space representation of the full controller (estimator + state feedback) that takes r and y as inputs and outputs u.

(d) Show that the open loop controller (with inputs r and y set to zero) has an eigenvalue in the right half plane.

Problem 2. The lateral dynamics of the vectored thrust aircraft example described in Example 7.9 can be obtained by considering the motion described by the states $z = (x, \theta, \dot{x}, \dot{\theta})$. Assume that we can only measure the position of the aircraft x, corrupted by white noise with intensity $R_w = 10^{-4}$.

(a) Construct an estimator for these dynamics by setting the eigenvalues of the observer into a *Butterworth pattern* with $\lambda_{\text{bw}} = -3.83 \pm 9.24i$, $-9.24 \pm 3.83i$. Show the response of the estimator starting from $\hat{z}(0) = (0.2, 0, 0, 0)$, assuming that the system remains at the origin but with noisy measurement of the x position.

(b) Construct an optimal estimator for the system assuming that the system is subject to white noise disturbances v with intensity $R_v = 0.1$ applied at the system input. Compute the response of the estimator as in part (a) and compare the initial condition response to that obtained in part (a).

(c) For each of the estimators above, combine them with the state space controller computed in Example 7.9 in FBS2e with $\rho = 1$ and plot the step response of the closed loop system in the presence of noisy measurements.

The following problem is a conceptual problem that you should be able to do without using the textbook, your notes, or the Internet. It is representative of the type of problem you can expect on the (closed-book) final exam (so you might want to first attempt solving it without using any notes).

Problem 3. Consider a system under a coordinate transformation $z = Tx$, where $T \in \mathbb{R}^{n \times n}$ is an invertible matrix.

(a) Show that the observability matrix for the transformed system is given by $\widetilde{W}_\text{o} = W_\text{o} T^{-1}$.

(b) Use this fact to show that the observability of a system is independent of the choice of state space coordinates.