CALIFORNIA INSTITUTE OF TECHNOLOGY
Computing and Mathematical Sciences

CDS 110/ChE 105

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Problem 1. The human body is a complex biological machine consisting of many interconnected control systems. One such system is the control of blood glucose level, which is described by Figure 4.19 and the subsection "Insulin–Glucose Dynamics" in FBS2e, Section 4.6. Models of varying complexity have been created, but we will consider a very simplified control scheme shown in the diagram below. The following questions do not require a detailed understanding of the biological mechanisms outlined above but rather a high-level understanding of the underlying control scheme.

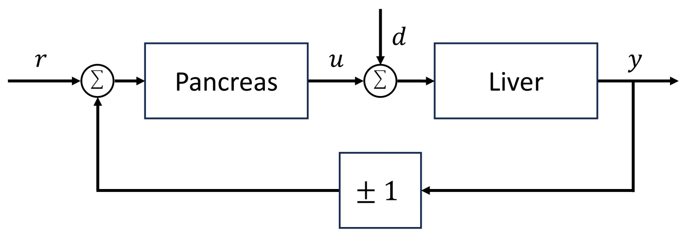

Figure 1: Blood glucose control scheme. Here, r is the desired blood glucose level, u are chemicals secreted into the blood by the pancreas, and y is the output blood glucose level after the liver's response to the secreted chemicals. When the glucose level is high, beta cells in the pancreas secrete insulin, which directs the liver to process excess glucose in the blood supply. When the glucose level is low, alpha cells in the pancreas secrete glucagon, which directs the liver to release its glucose stores. In the shown diagram, d represents disturbances to the secreted chemicals, which may result from causes such as eating, exercise, stress, and more.

(a) Type 1 diabetes is an autoimmune disease where the pancreas's beta cells are destroyed by the immune system. Despite the remaining beta cells working harder to make insulin (to compensate for destroyed cells), the pancreas secretes insufficient insulin. Which of the following functions of the pancreas is *most directly* compromised in this scenario: sensing, computation, or actuation? Provide a short explanation of why.

(b) Type 2 diabetes is a disease where the liver is unable to absorb insulin or use it as effectively, which may be in part due to a reduction in insulin receptors on the surface of cells in the liver. As a result, the liver does not process excess glucose effectively. Which of the following functions of the liver is *most directly* compromised in this scenario: sensing, computation, or actuation? Provide a short explanation of why.

(c) Artificial pancreases are devices that continuously monitor blood glucose levels and secrete insulin and/or glucagon to help regulate glucose in individuals with type 1 diabetes. Consider a hypothetical scenario where one such artificial pancreas is given to an individual, perhaps to someone in the UK (see https://tinyurl.com/nhs-artificial-pancreas). Suppose that there is build-up of salts in the tubing that delivers insulin to the individual, and as a result, the individual's blood glucose level is not effectively regulated (thankfully, they receive a cell phone notification and immediately seek medical care). Which of the following functions of the artificial pancreas is *most directly* compromised in this scenario: sensing, computation, or actuation? Provide a short explanation of why.

Problem 2. Consider an open loop system with the nonlinear input/output relation $y = F(u)$. Assume that the system is closed with the proportional controller $u = -k(y - r)$. Show that the input/output relation of the closed loop system is

$$y + \frac{1}{k} F^{-1}(y) = r.$$

Estimate the largest deviation from ideal linear response $y = r$. Illustrate by plotting the input output responses for a) $F(u) = \sqrt{u}$ and b) $F(u) = u^2$ with $0 \leq u \leq 1$ and $k = 5$, 10, and 100.

Problem 3. Let $x \in \mathbb{R}$ and $u \in \mathbb{R}$. Solve the differential equation

$$\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + y = 2\frac{du}{dt} + u.$$

Determine the responses to a unit step $u(t) = 1$ and the exponential signal $u(t) = e^{st}$ when the initial conditions are zero.

Note: exponential signals are described in more detail in FBS Section 2.2. They are one of the ways to obtain the "transfer function" for a linear system, which we will study later in the term (but which you can read about in FBS, Chapter 2).

Problem 4. In this problem we will build a model of the MinSeg system that we saw in class and design a controller to stabilize the upright position.

This problem is given in Google Colab notebook minseg-propctrl.ipynb. You should make a copy of this template, edit it to answer the questions in the notebook, and then submit a PDF of the edited version of the notebook as your solution to this problem. More directions are provided in the Colab notebook.