CDS 101/110: Lecture 8-1
PID Control

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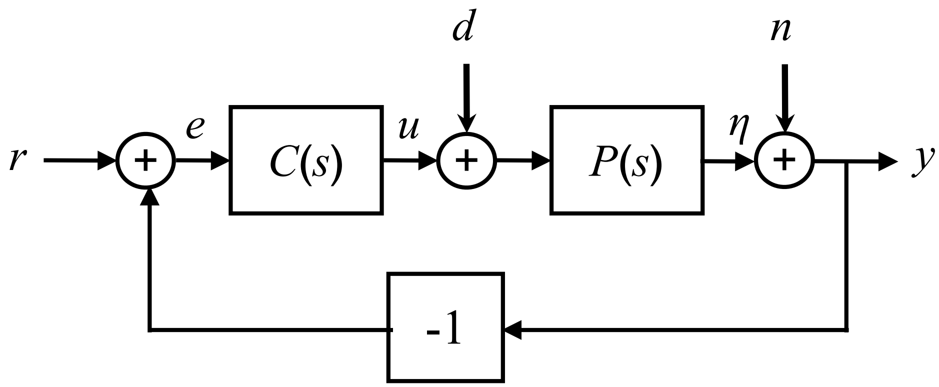
Goals:

- Show how to use “loop shaping” using PID to achieve a performance specification
- Discuss the use of integral feedback and anti-windup compensation

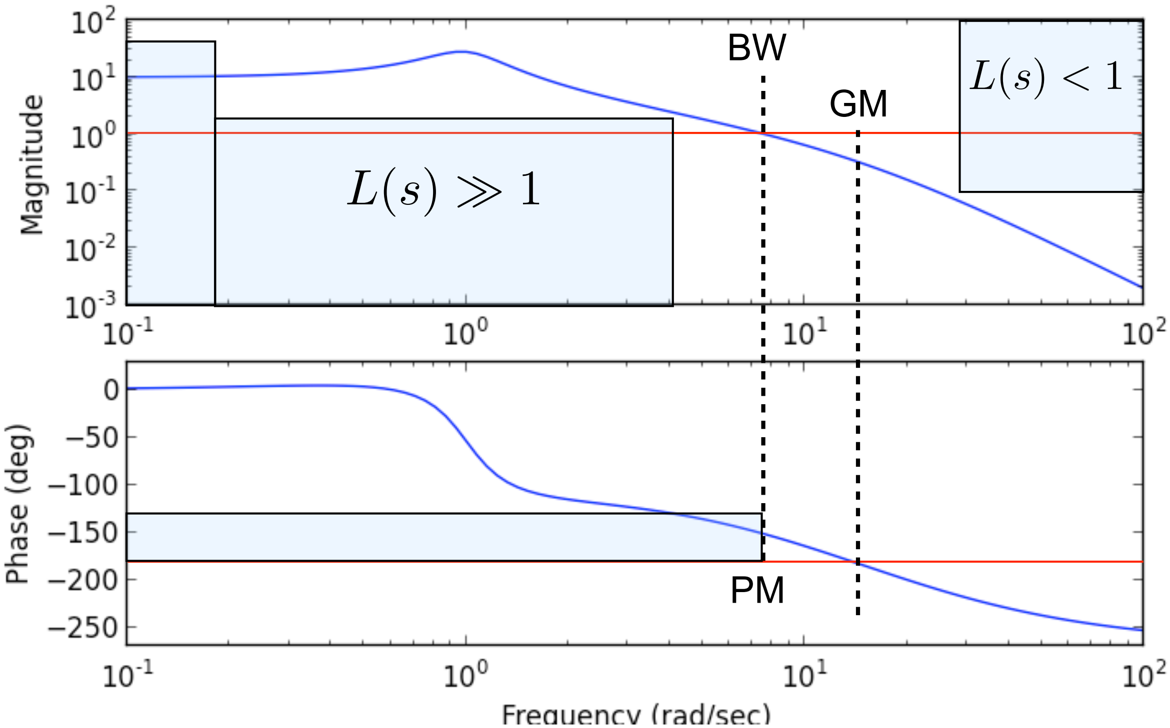
Reading:

- Åström and Murray, *Feedback Systems*, Ch 11

"Loop Shaping": Design Loop Transfer Function



$$H_{er} = \frac{1}{1+L} \qquad\qquad H_{\eta n} = \frac{-L}{1+L}$$



Translate specs to "loop shape"

$$L(s) = P(s)C(s)$$

- Design C(s) to obey constraints

Typical loop constraints

- High gain at low frequency
  - Good tracking, disturbance rejection at low freqs
- Low gain at high frequency
  - Avoid amplifying noise
- Sufficiently high bandwidth
  - Good rise/settling time
- Shallow slope at crossover
  - Sufficient phase margin for robustness, low overshoot

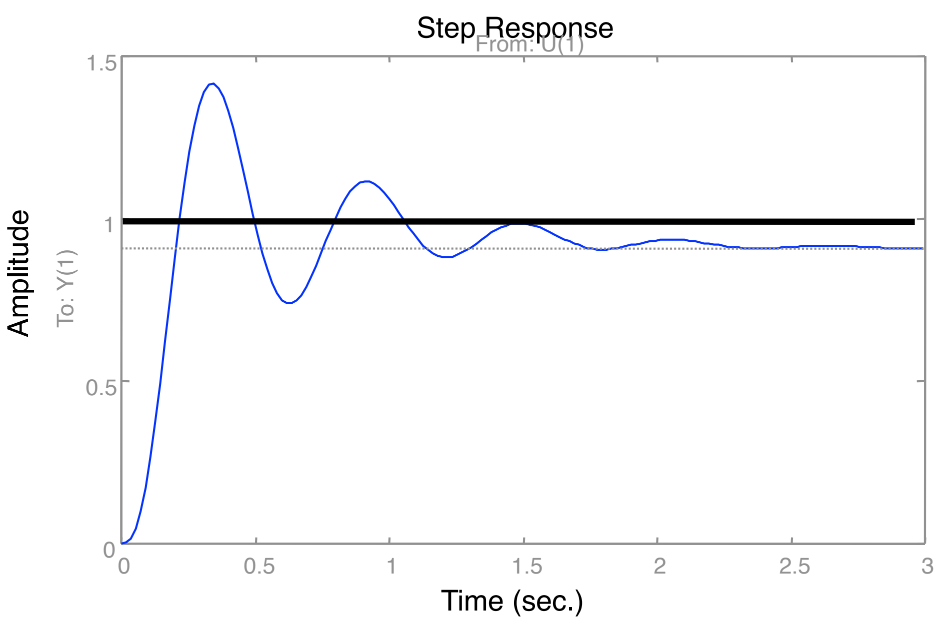
Key constraint: slope of gain curve determines phase curve

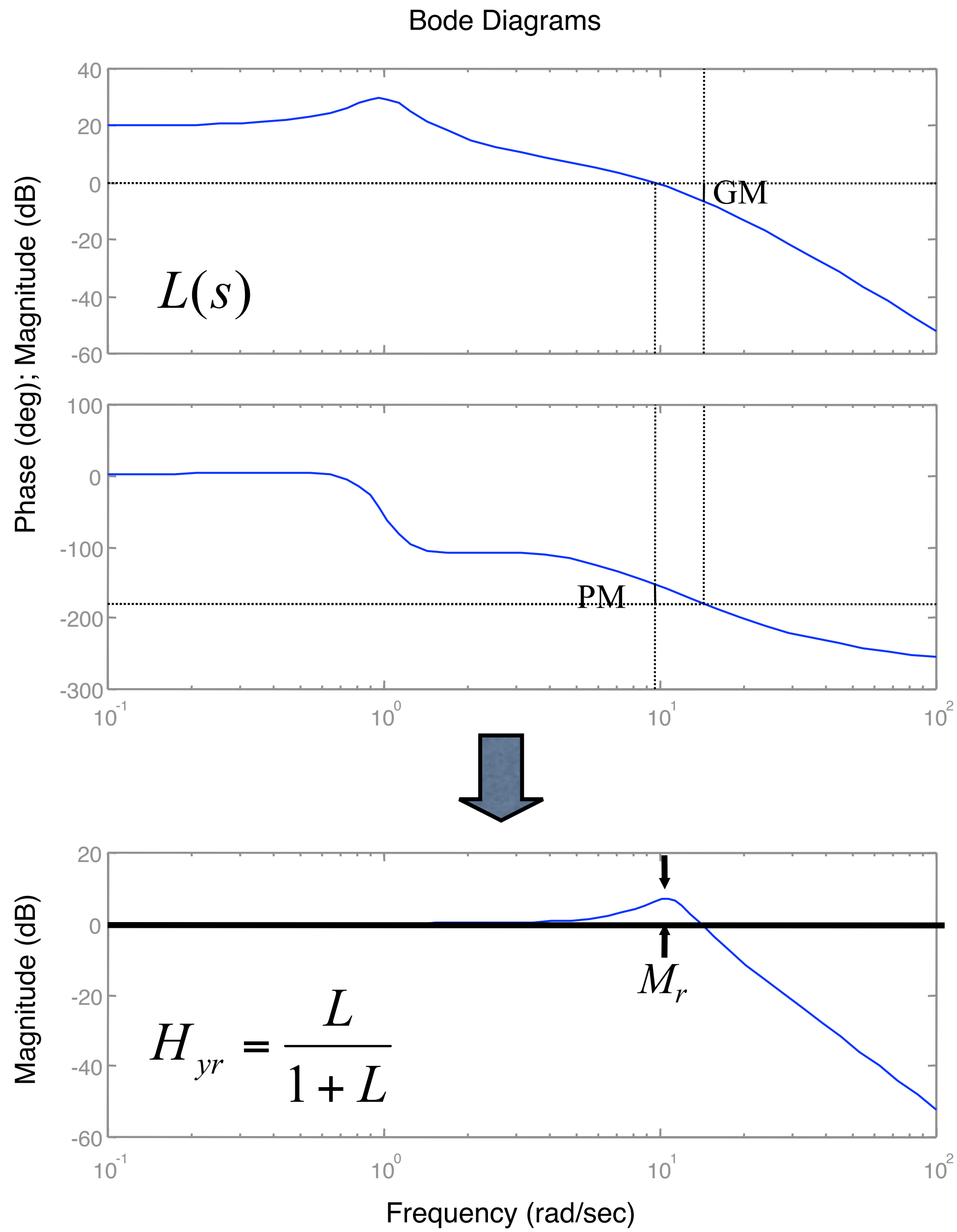
- Can't independently adjust
- Eg: slope at crossover sets PM

Gain Crossover Specifications

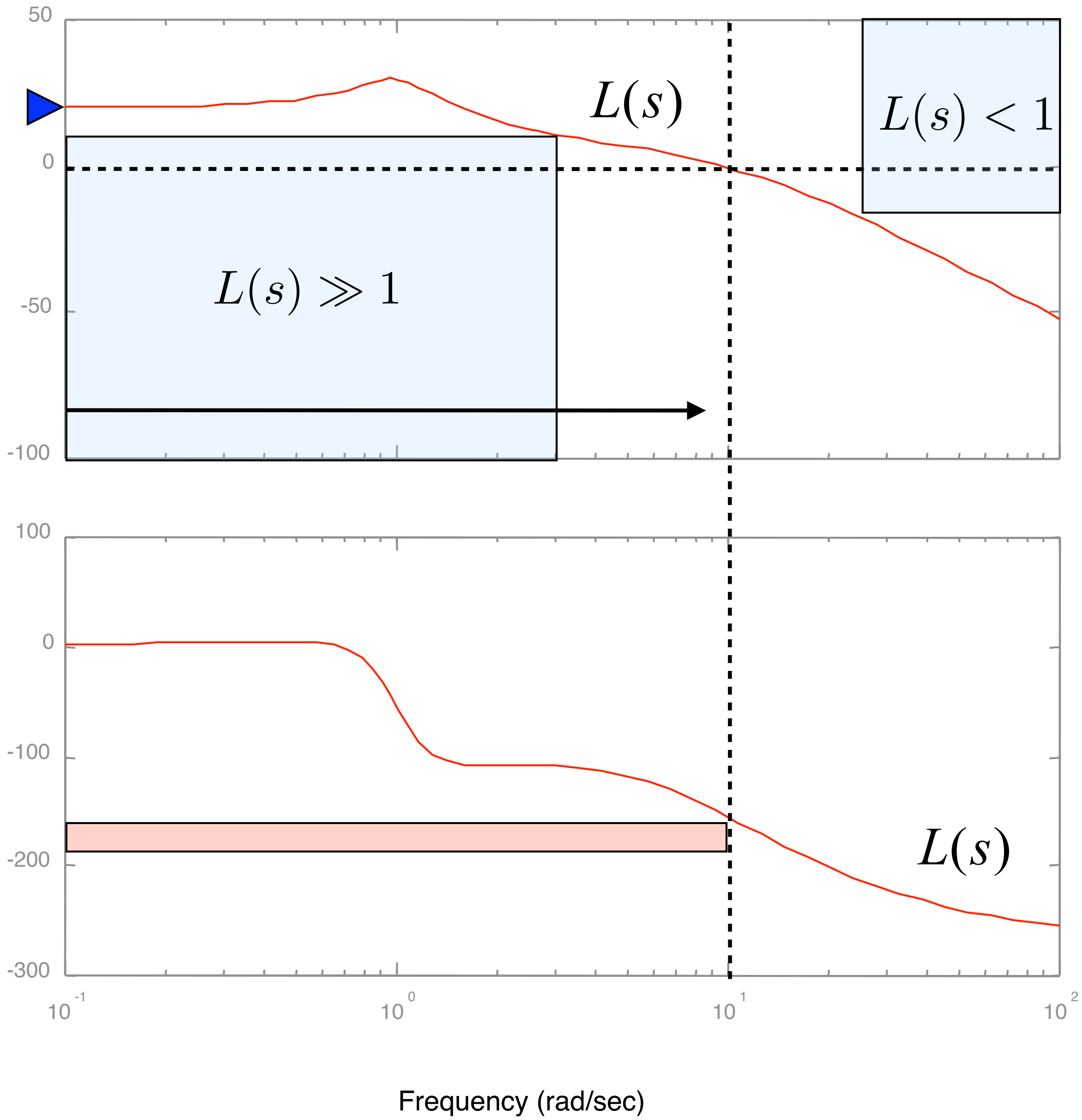
High phase at gain crossover can lead to poor performance

- System can be stable but still have bad response at certain frequencies
- Typically occurs if system has low phase margin ⇒ get resonant peak in closed loop (Mr) + poor step response
- Solution: specify minimum phase margin. Typically 45˚ or more





Overview of Loop Shaping



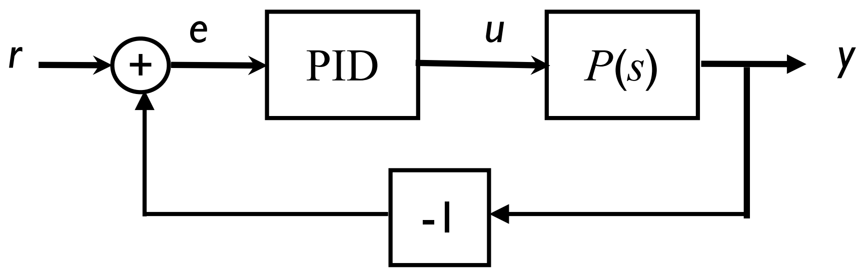
Performance specification

- ▶ Steady state error
- ☐ Tracking error
- → Bandwidth (possibly from rise/settling time)
- ☐ Robustness margins/ overshoot

Approach: "shape" loop transfer function using C(s)

- P(s) + specifications given
- L(s) = P(s) C(s)
  - Use C(s) to choose desired shape for L(s)
- Important: can't set gain and phase independently
  - Slope at crossover determines phase

Overview: PID control



$$u = k_p e + k_i \int e\,dt + k_d \dot{e}$$

Intuition

- Proportional term: provides inputs that correct for "current" errors
- Integral term: insures steady state error goes to zero
- Derivative term: provides "anticipation" of upcoming changes

A bit of history on "three term control"

- First appeared in 1922 paper by Minorsky: "Directional stability of automatically steered bodies" under the name "three term control"
- Also realized that "small deviations" (linearization) could be used to understand the (nonlinear) system dynamics under control

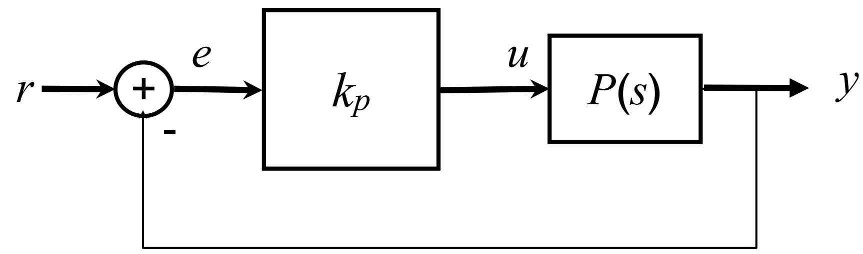
Utility of PID

- PID control is most common feedback structure in engineering systems
- For many systems, only need PI or PD (special case)
- Many tools for tuning PID loops and designing gains

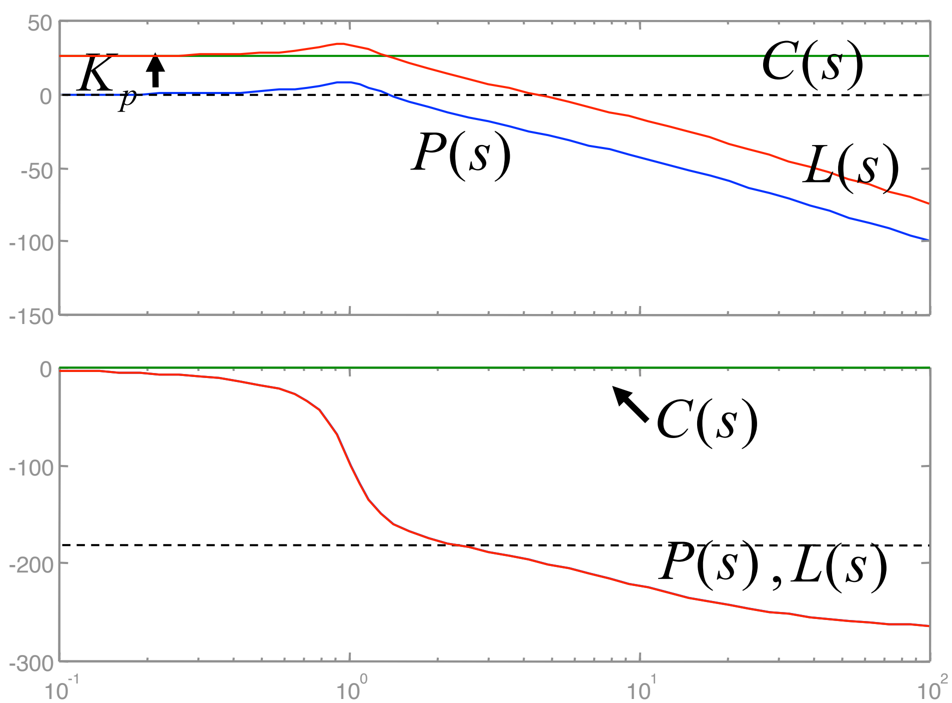
Proportional Feedback

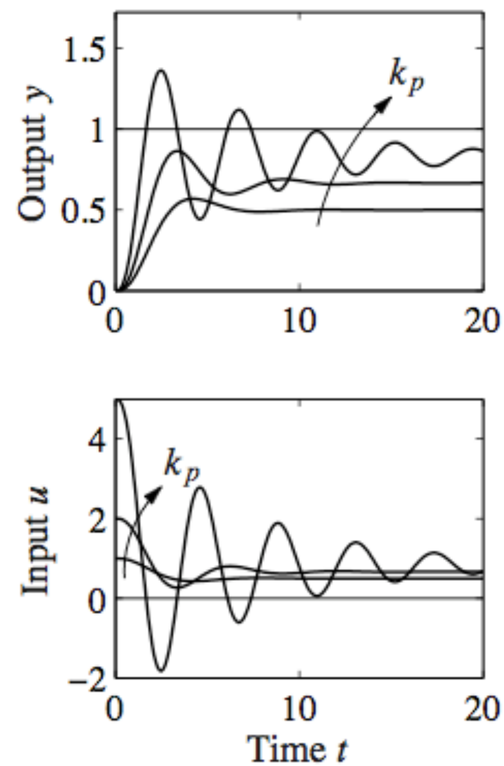
Simplest controller choice: $u = k_p e$

- Effect: lifts gain with no change in phase
- Good for plants with low phase up to desired bandwidth
- Bode: shift gain up by factor of kp
- Step response: better steady state error, but with decreasing stability



$k_p > 0$

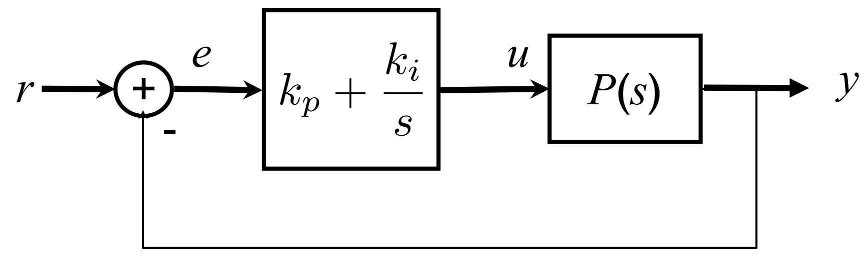




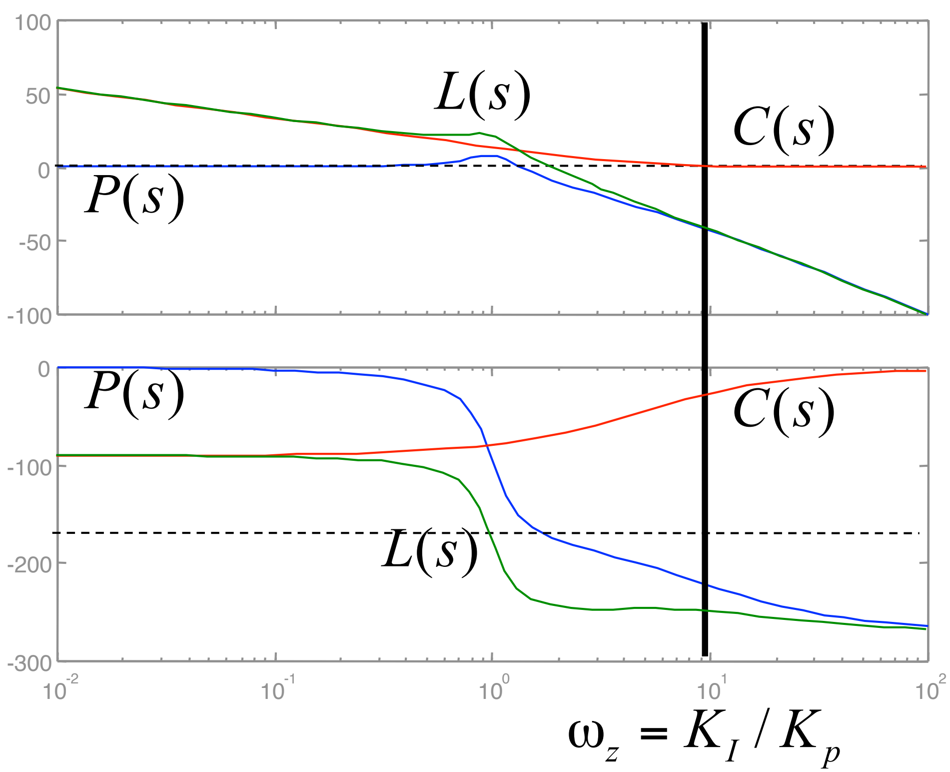
Proportional + Integral Compensation

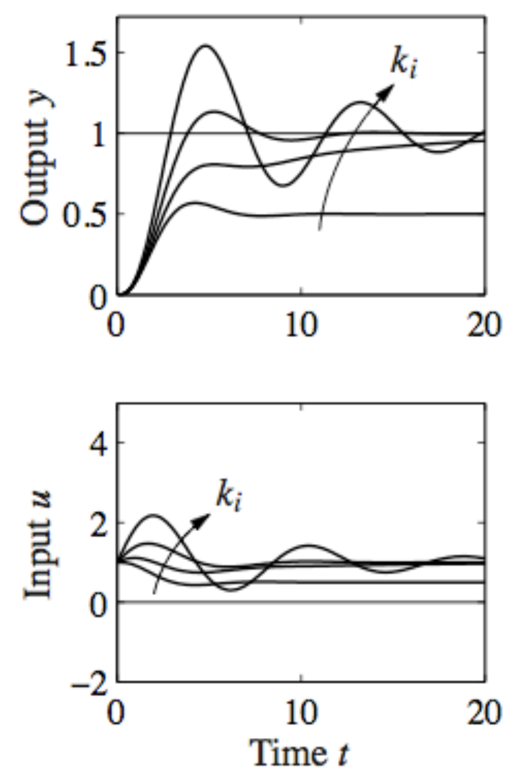
Use to eliminate steady state error

- Effect: lifts gain at low frequency
- Gives zero steady state error
- Bode: infinite SS gain + phase lag
- Step response: zero steady state error, with smaller settling time, but more overshoot

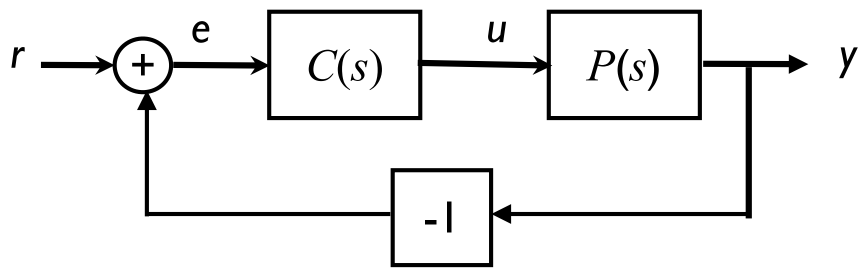


$$k_p > 0, \quad k_i > 0$$

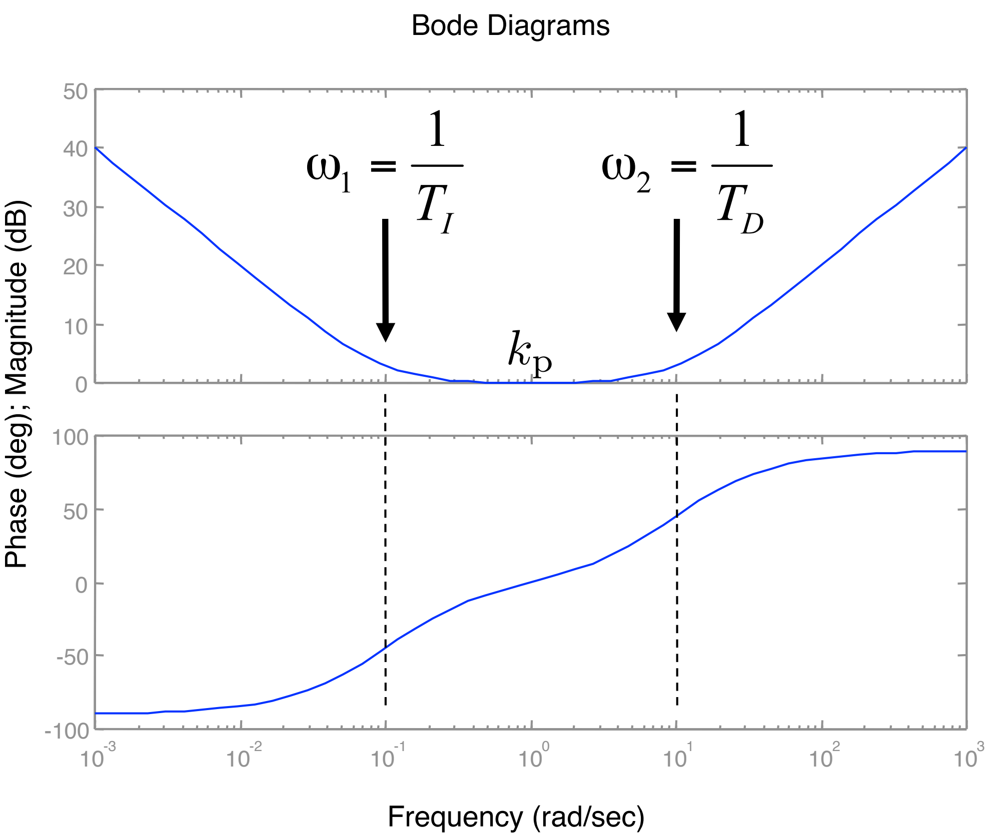


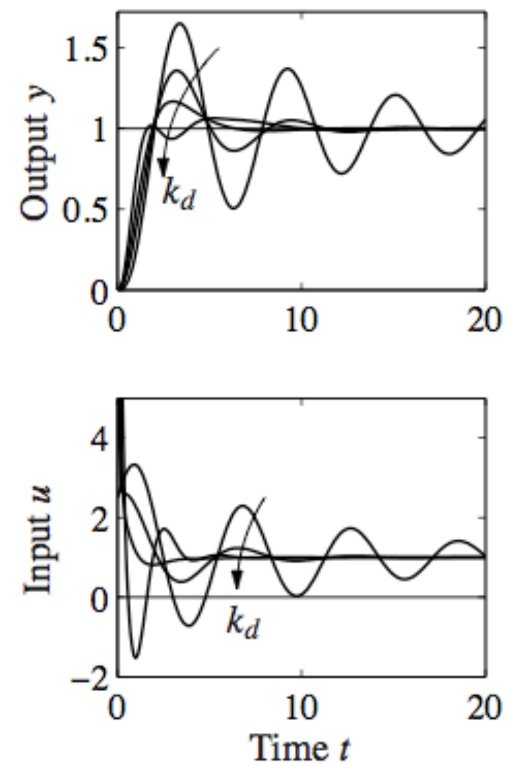


Proportional + Integral + Derivative (PID)



$$
\begin{aligned}
C(s) &= k_\text{p} + \frac{k_\text{i}}{s} + k_\text{d} s \\
&= k_\text{p}\left(1 + \frac{1}{T_\text{i} s} + T_\text{d} s\right) \\
&\approx T_\text{d} k_\text{p} \frac{(s + 1/T_\text{i})(s + 1/T_\text{d})}{s}
\end{aligned}
$$





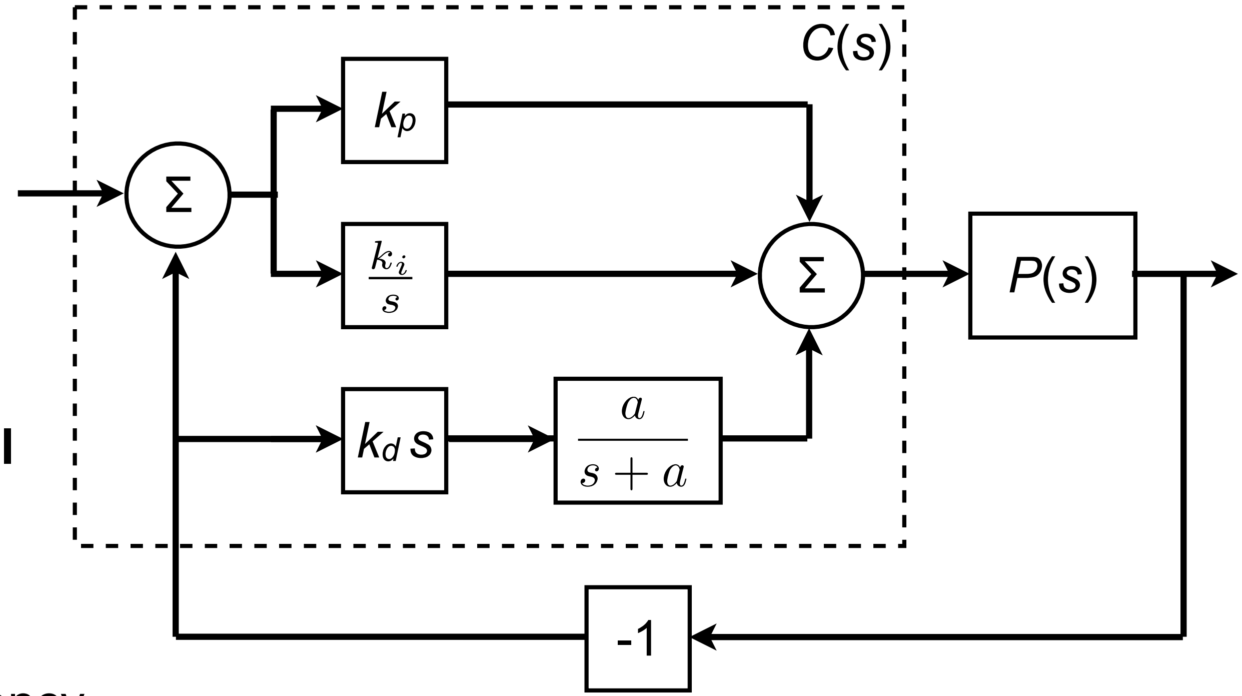
Implementing Derivative Action

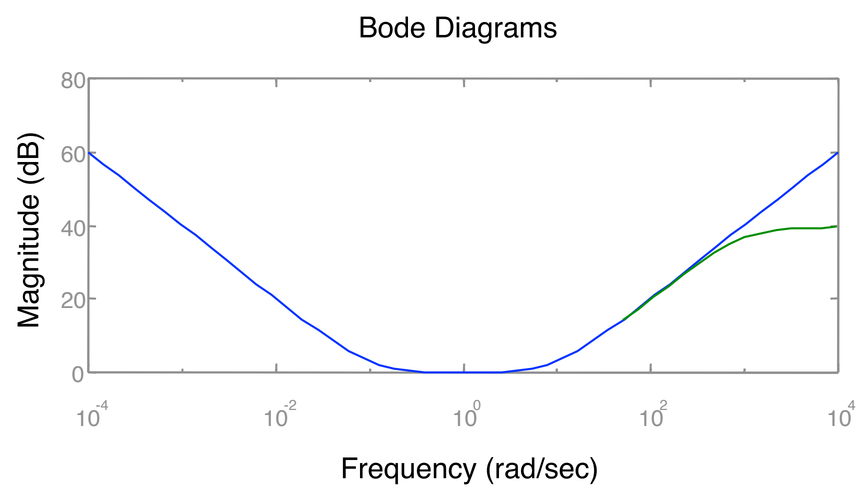
Problems with derivatives

- High frequency noise amplified by derivative term
- Step inputs in reference can cause large inputs
- Show up in Gang of Four...

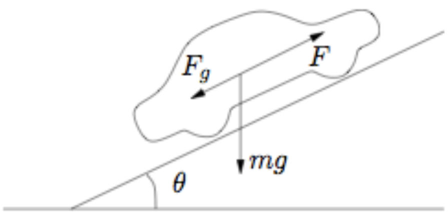
Solution: modified PID control

- Use high frequency rolloff in derivative term
  - first order filter will give finite gain at high frequency
  - use higher order filter if needed
- Don't feed reference signal through derivative block
  - Useful when reference has unwanted high frequency content
  - Better solution: reference shaping via two DOF design (F(s) block)
- Many other variations (see AM08 + refs)

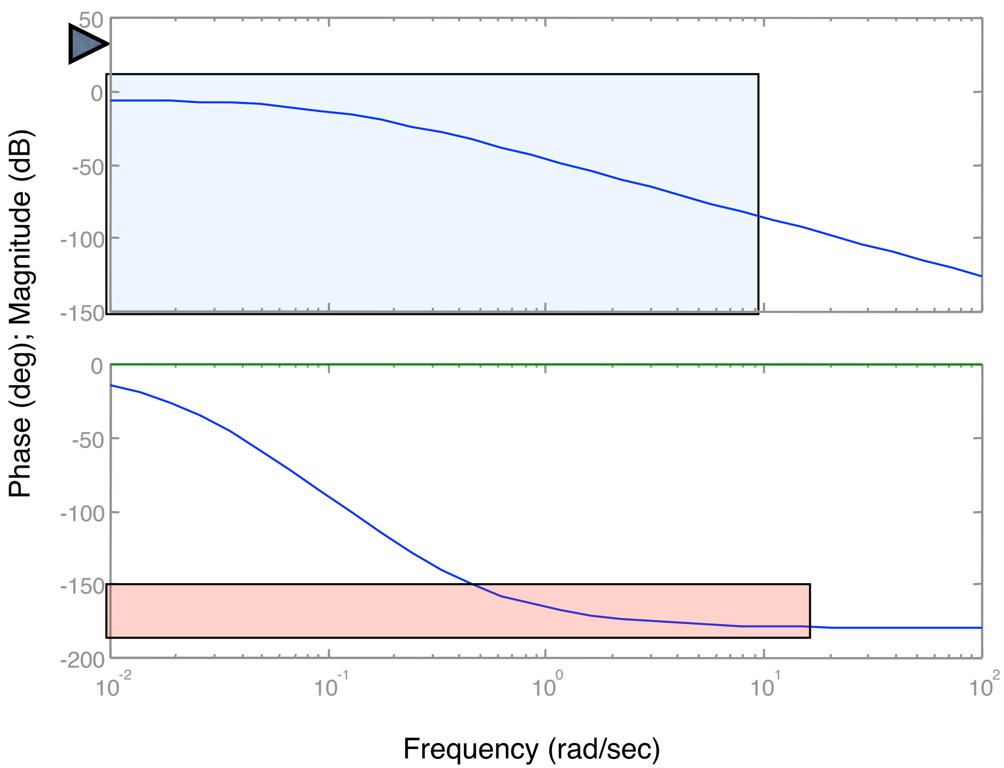




Example: Cruise Control using PID - Specification



$$P(s) = \frac{1/m}{s + b/m} \times \frac{r}{s + a}$$



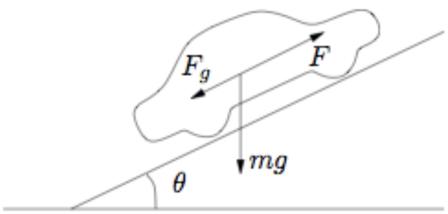
Performance Specification

- ≤ 1% steady state error
  - Zero frequency gain > 100
- ≤ 10% tracking error up to 10 rad/sec
  - Gain > 10 from 0-10 rad/sec
- ≥ 45˚ phase margin
  - Gives good relative stability
  - Provides robustness to uncertainty

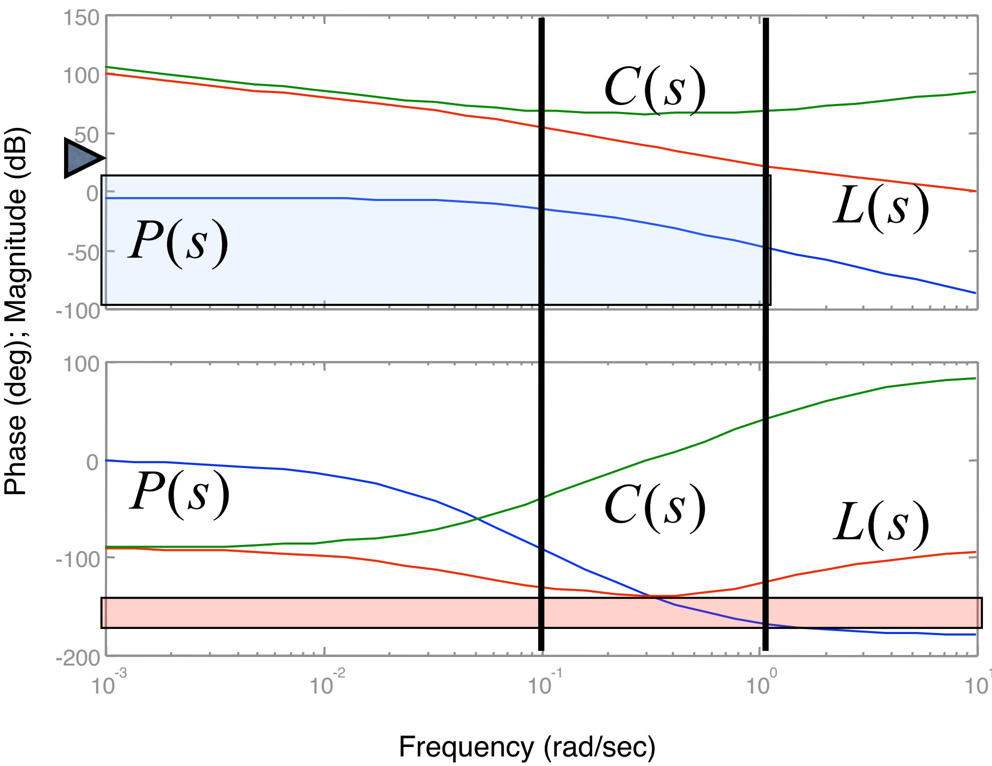
Observations

- Purely proportional gain won't work: to get gain above desired level will not leave adequate phase margine
- Need to increase the phase from ~0.5 to 2 rad/sec and increase gain as well

Example: Cruise Control using PID - Design



$$P(s) = \frac{1/m}{s + b/m} \times \frac{r}{s+a}$$



Approach

- Use integral gain to make steady state error small (zero, in fact)
- Use derivative action to increase phase lead in the cross over region
- Use proportional gain to give desired bandwidth

Controller

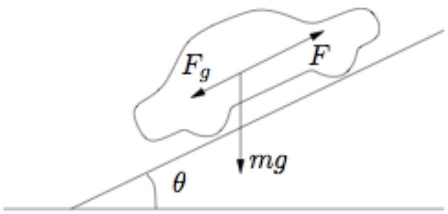
- Ti = 1/0.1; Td = 1/1; k = 2000

$$C(s) = 2000\frac{s^2 + 1.1s + 0.1}{s} = 2200 + \frac{200}{s} + 2000s$$

Closed loop system

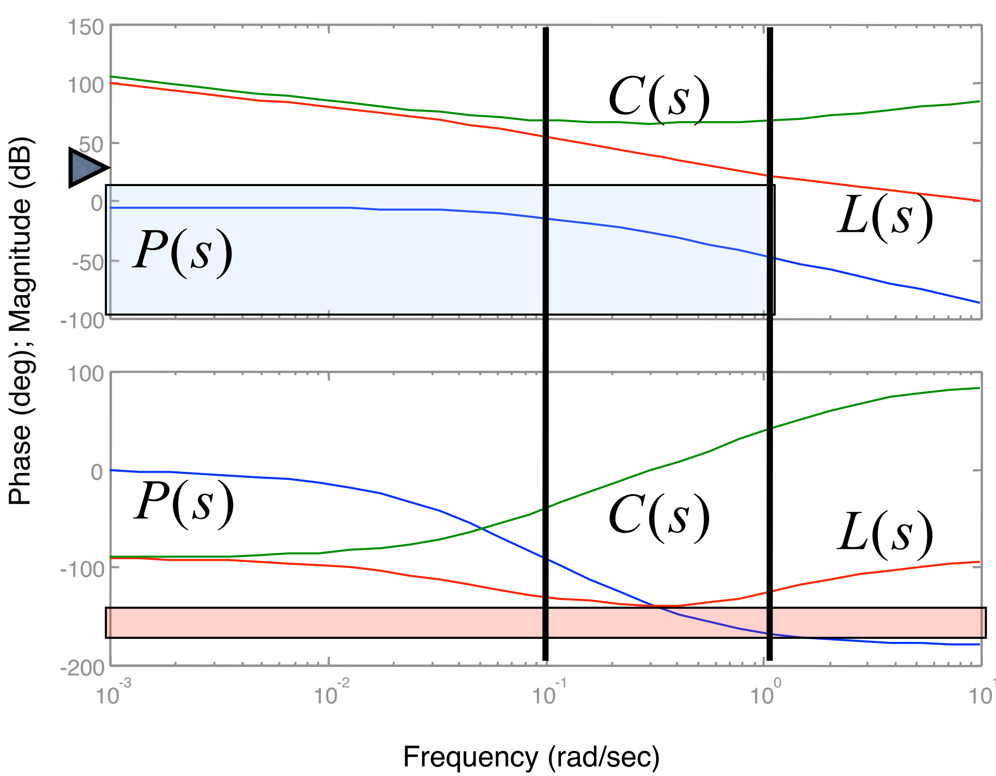
- Very high steady state gain
- Adequate tracking @ 1 rad/sec
- ~80° phase margin
- Verify with Nyquist

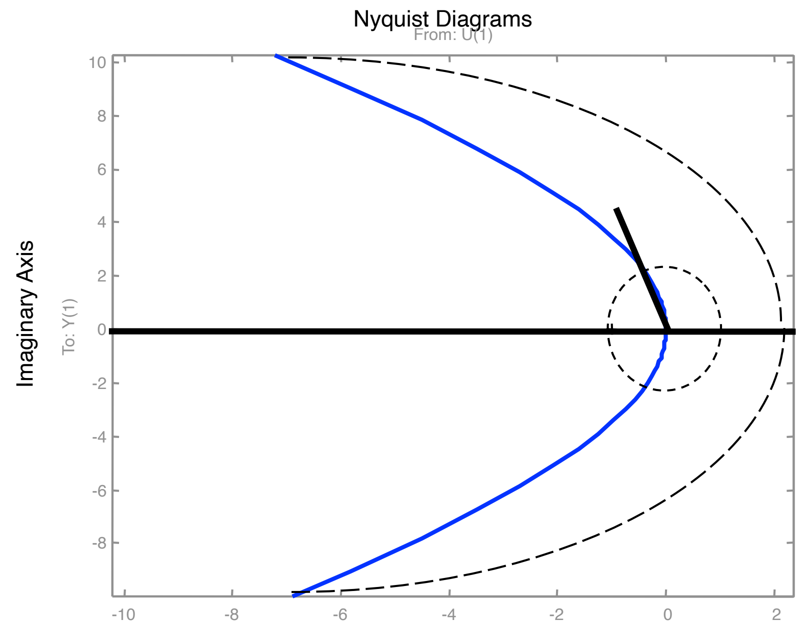
Example: Cruise Control using PID - Verification

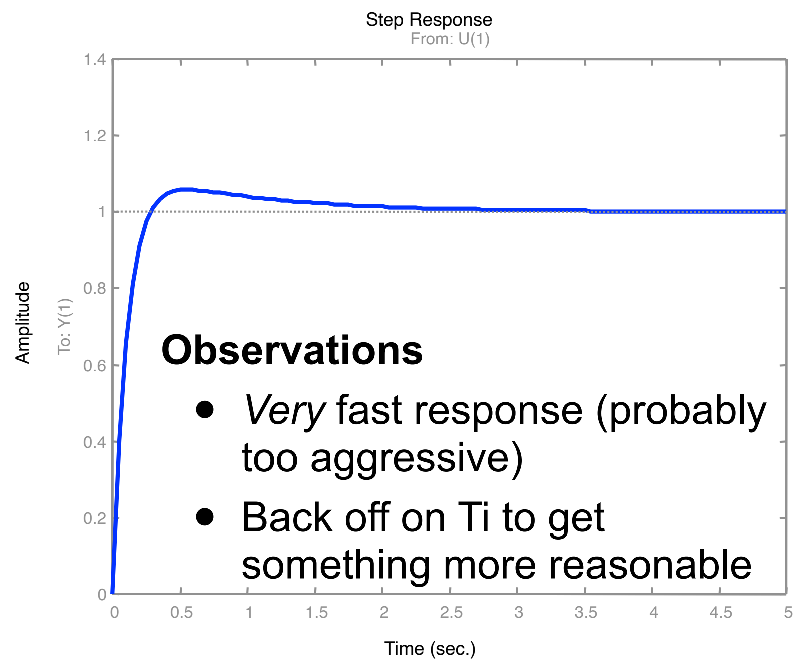


$$P(s) = \frac{1/m}{s + b/m} \times \frac{r}{s + a}$$

$$C(s) = 2000 \frac{s^2 + 1.1s + 0.1}{s}$$



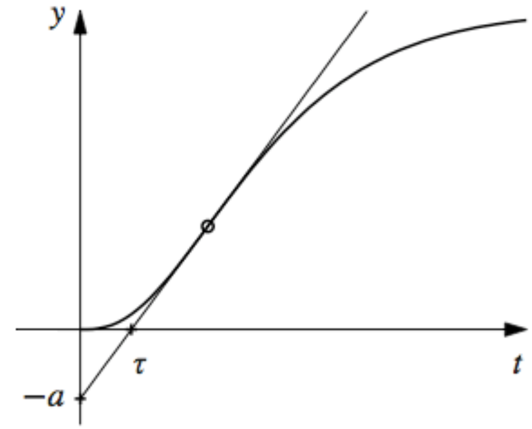




Observations

- *Very* fast response (probably too aggressive)
- Back off on Ti to get something more reasonable

PID Tuning

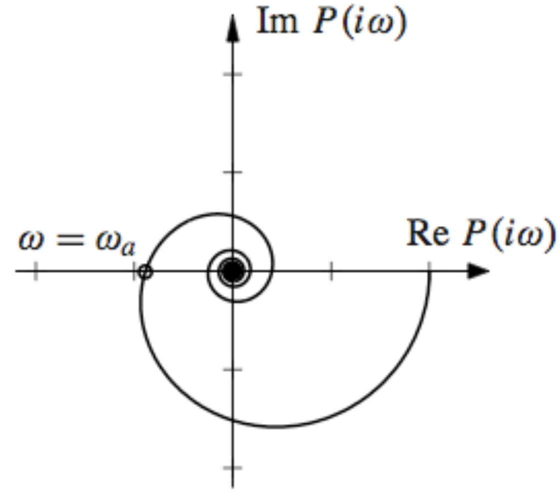


Zeigler-Nichols step response method

- Design PID gains based on step response
- Measure maximum slope + intercept
- Works OK for many plants (but underdamped)
- Good way to get a first cut controller

Ziegler-Nichols frequency response method

- Increase gain until system goes unstable
- Use critical gain and frequency as parameters



Variations

- Modified formulas (see text) give better response
- Relay feedback: provides automated way to obtain critical gain, frequency

Type	k_p	T_i	T_d
P	1/a		
PI	0.9/a	3τ	
PID	1.2/a	2τ	0.5τ

(a) Step response method

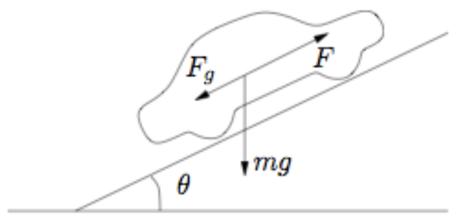
Type	k_p	T_i	T_d
P	$0.5k_c$		
PI	$0.4k_c$	$0.8T_c$	
PID	$0.6k_c$	$0.5T_c$	$0.125T_c$

(b) Frequency response method

$$k_p = \frac{0.15\tau + 0.35T}{K\tau} \quad \left(\frac{0.9T}{K\tau}\right), \qquad k_i = \frac{0.46\tau + 0.02T}{K\tau^2} \quad \left(\frac{0.3T}{K\tau^2}\right),$$

$$k_p = 0.22k_c - \frac{0.07}{K} \quad \left(0.4k_c\right), \qquad k_i = \frac{0.16k_c}{T_c} + \frac{0.62}{KT_c} \quad \left(\frac{0.5k_c}{T_c}\right).$$

Example: PID cruise control



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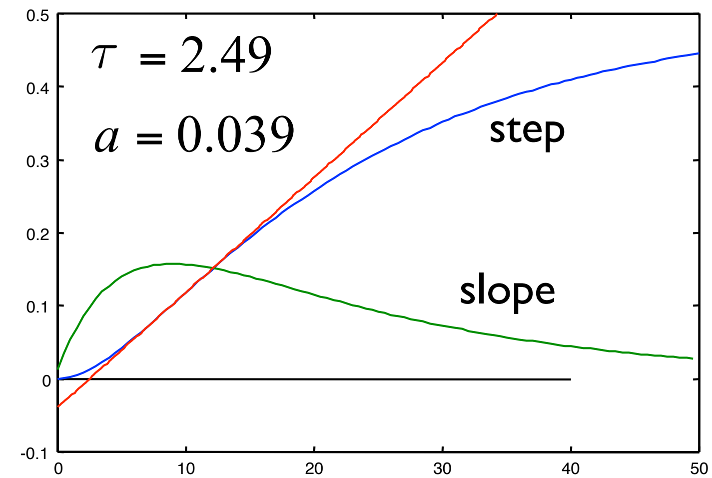
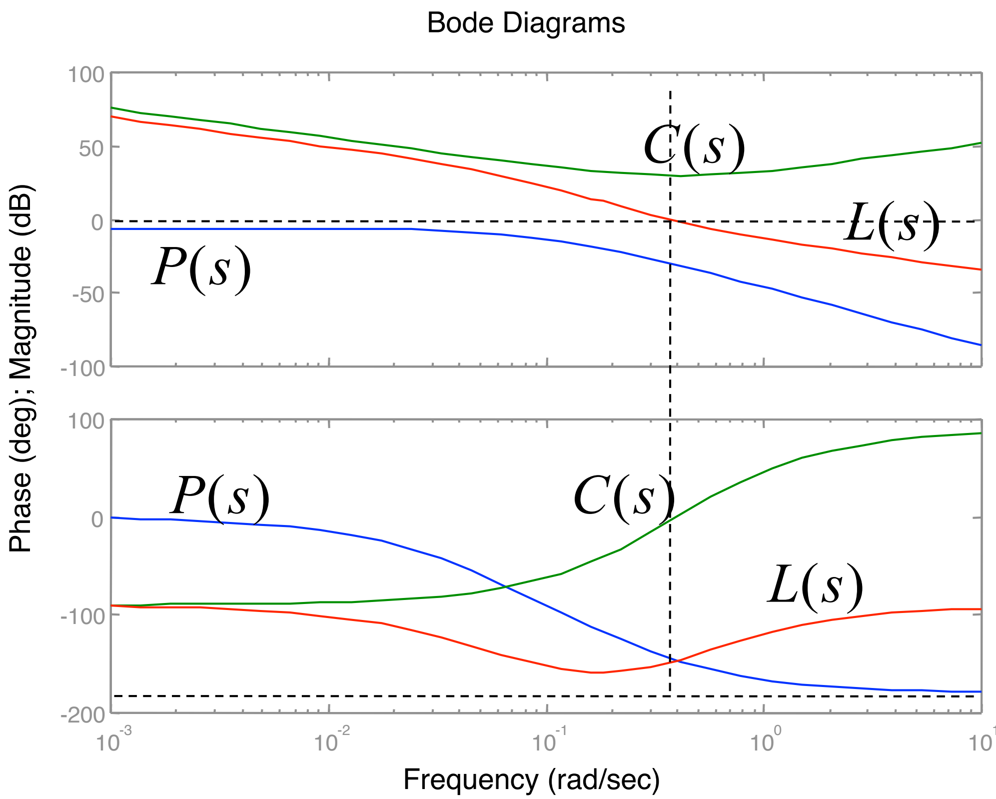
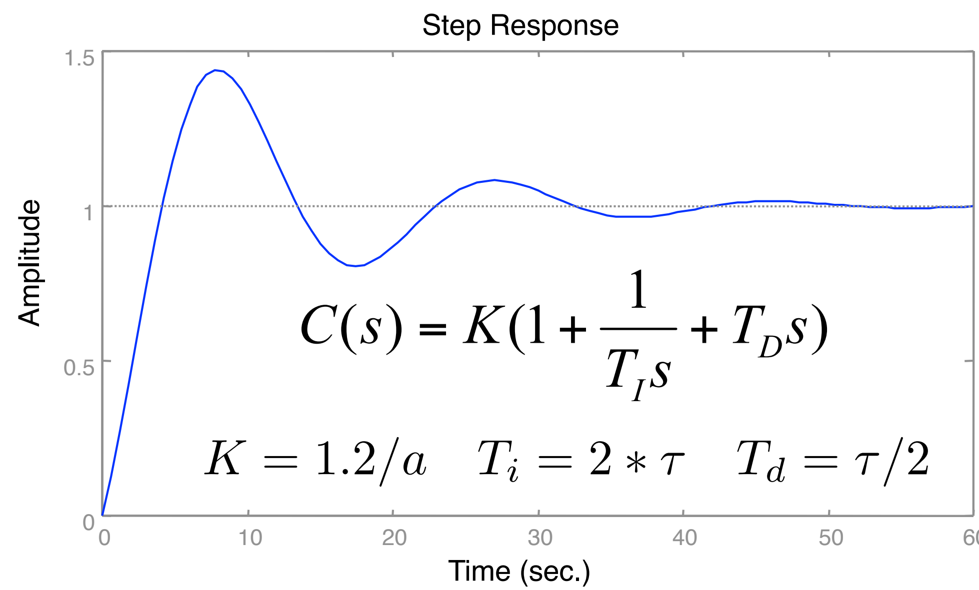


Ziegler-Nichols design for cruise controller

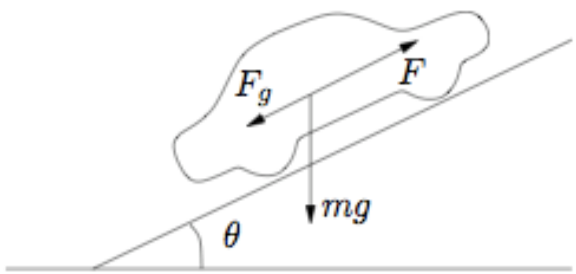
- Plot step response, extract τ and a, compute gains

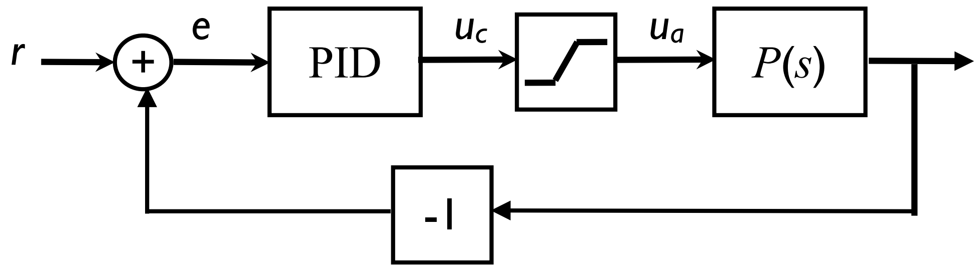




- Result: *sluggish* ⇒ increase loop gain + more phase margin (shift zero)

Windup and Anti-Windup Compensation



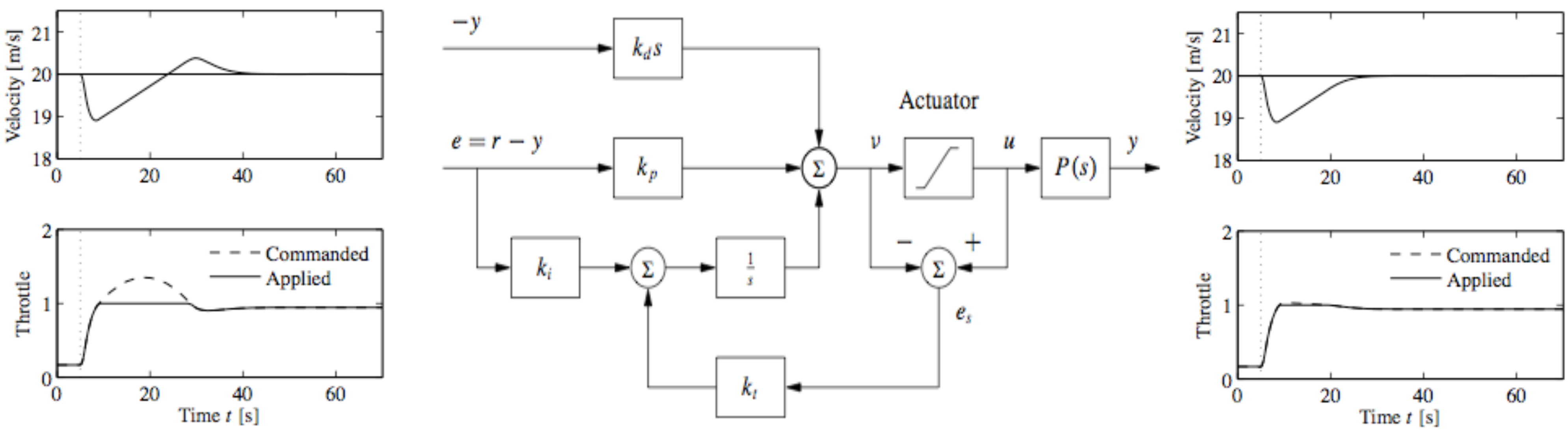


Problem

- Limited magnitude input (saturation)
- Integrator "winds up" => overshoot

Solution

- Compare commanded input to actual
- Subtract off difference from integrator



(a) Windup

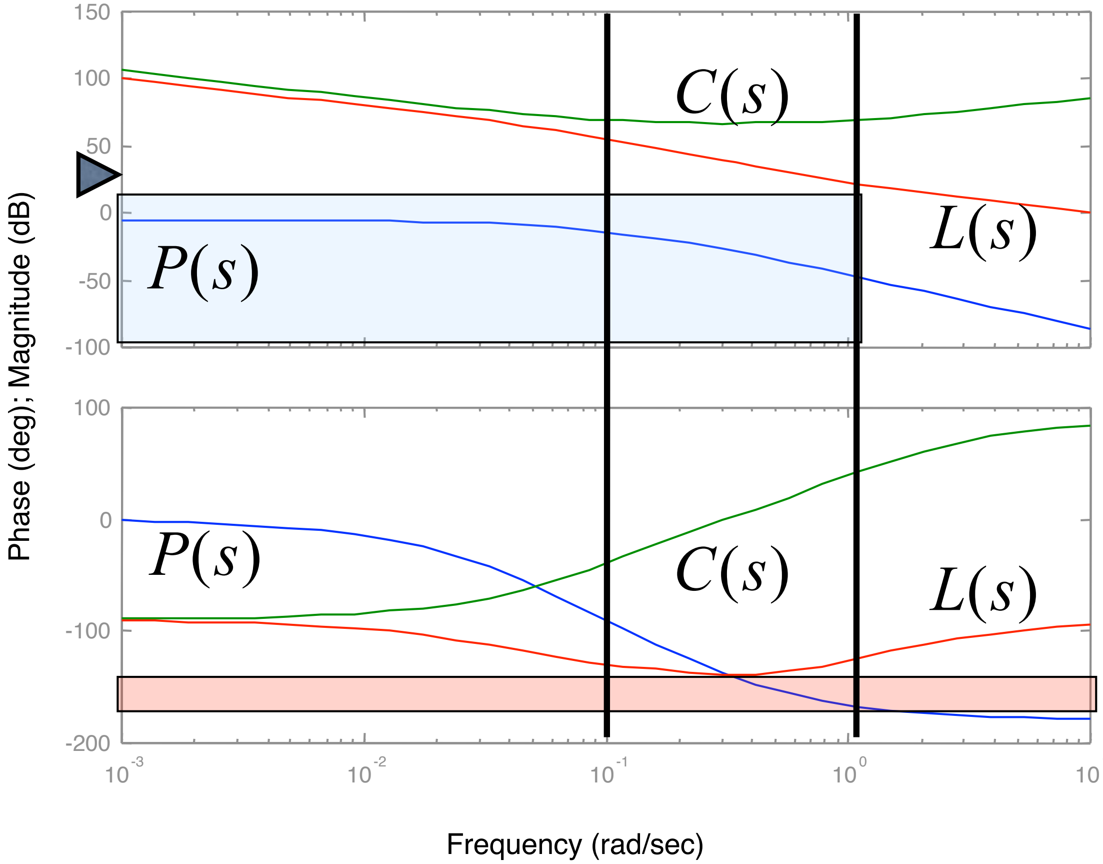
(b) Anti-windup

Summary: Frequency Domain Design using PID

Loop Shaping for Stability & Performance

- Steady state error, bandwidth, tracking

$$H_{ue}(s) = K_p + K_I \times \frac{1}{s} + K_D s$$



Main ideas

- Performance specs give bounds on loop transfer function
- Use controller to shape response
- Gain/phase relationships constrain design approach
- Standard compensators: proportional, PI, PID