# L8-3: Magnetic levitation example

**CDS 110/ChE 105, Winter 2024**
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This notebook contains the code used to create the magnetic levitation example in Lecture 8-1.

In [1]:
```
import numpy as np
import scipy as sp
import matplotlib.pyplot as plt

from math import pi

try:
  import control as ct
  print("python-control", ct.__version__)
except ImportError:
  !pip install control
  import control as ct

import control.optimal as opt
import control.flatsys as fs
```

```
python-control 0.10.0
```

The magnetic leviation system is governing by following equation:

$$\ddot{z} = g - \frac{k_m k_A^2}{m} \frac{u^2}{z^2} - \frac{c}{m} \dot{z}$$

and the system's output $v_{ir}$ is the following:

$$v_{ir} = k_T z + v_0$$

In [2]:
```
# System dynamics
maglev_params = {
    'kT': 613.65,    # gain between position and voltage
    'v0': -16.18,          # voltage offset at zero position
    'm': 0.2,          # mass of ball, kg
    'g': 9.81,       # gravitational constant
    'kA': 1,           # electromagnet conductance
    'c': 1           # damping (added to improve visualization)
}
# gain on magnetic attractive force
maglev_params['km'] = 3.13e-3 * (maglev_params['m']/2) / maglev_params['kA']

def maglev_update(t, x, u, params):
    m, g, kA, km, c = map(params.get, ['m', 'g', 'kA', 'km', 'c'])
    return np.array([
```

```python
        x[1],
        g - km/m * (kA * u[0])**2 / x[0]**2 - c * x[1]
    ])

def maglev_output(t, x, u, params):
    kT, v0 = map(params.get, ['kT', 'v0'])
    return np.array([kT * x[0] + v0])

maglev = ct.nlsys(
    maglev_update, maglev_output, params=maglev_params, name='maglev',
    inputs='Vu', outputs='Vy', states=['pos', 'vel']
)
```

In [3]:

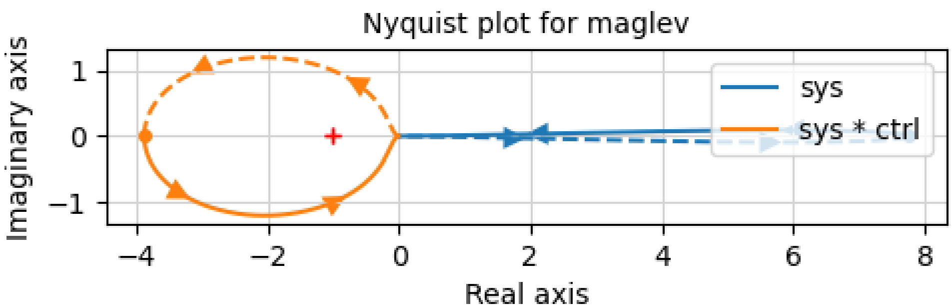
```python
# Compute the equilibrium point that holds the ball at the origin
xeq, ueq = ct.find_eqpt(maglev, [0.02, 0], 0.2, y0=0)
print(f"{xeq=}, {ueq=}", end='\n----\n')

# Compute the linearization at that point
magP = ct.linearize(maglev, xeq, ueq)
print(magP, end='\n----\n')

print("Poles:", magP.poles())
print("Zeros:", magP.zeros())
```

```
xeq=array([2.63668215e-02, 1.07930070e-25]), ueq=array([2.08754147])
----
<StateSpace>: sys[0]
Inputs (1): ['u[0]']
Outputs (1): ['y[0]']
States (2): ['x[0]', 'x[1]']

A = [[  0.            1.        ]
     [744.07466586   -1.        ]]

B = [[ 0.        ]
     [-9.39861794]]

C = [[613.65   0.  ]]

D = [[0.]]

----
Poles: [ 26.78231416+0.j -27.78231416+0.j]
Zeros: []
```

In [4]:

```python
# Controller (analog circuit)
k1 = 0.5                                    # gain set by gain pot
R1 = 22000                                  # Internal resistor
R2 = 22000                                  # Resistor plug-in
R = 2000; C = 1e-6                  # RC plug-in

# Controller based on analog circuit
magC1 = -ct.tf([(R1 + R) * C, 1], [R * C, 1]) * k1 * R2/R1
magL1 = magP * magC1
```

In [5]:
```
# Nyquist plot
plt.figure(figsize=[5, 3.75])
magP.name = "sys"
magL = magP * magC1
magL.name = "sys * ctrl"
ct.nyquist_plot([magP, magL])
plt.gca().set_aspect('equal')
plt.suptitle("")
plt.title("Nyquist plot for maglev", fontsize=10)
plt.tight_layout()
```

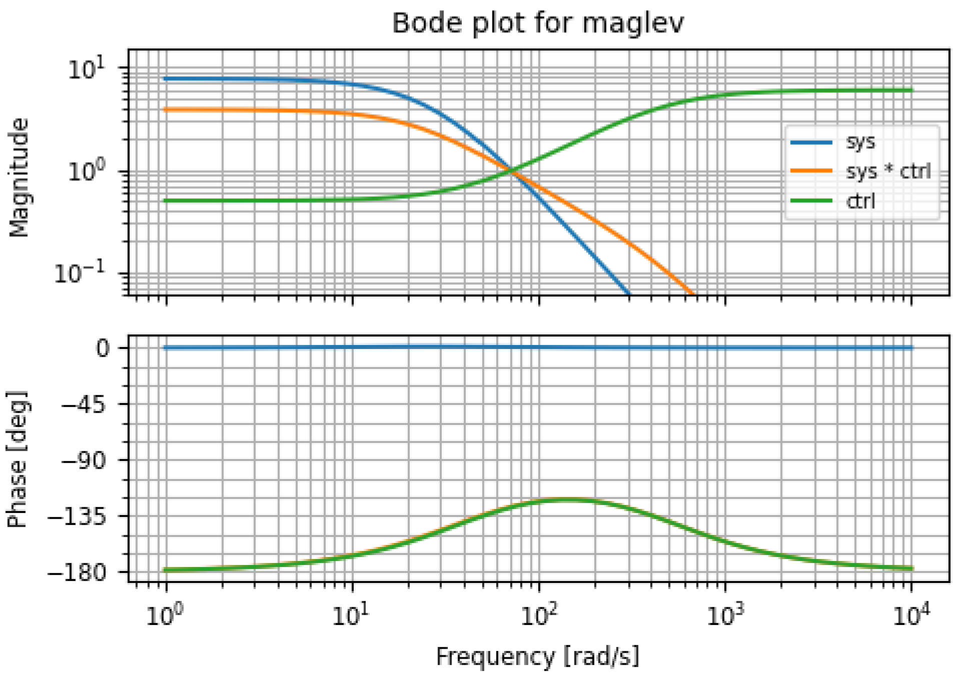



make some comment about how gain can be increased infinitely but there is a lower limit for stability

also plot/calculate gain/phase/stability margins

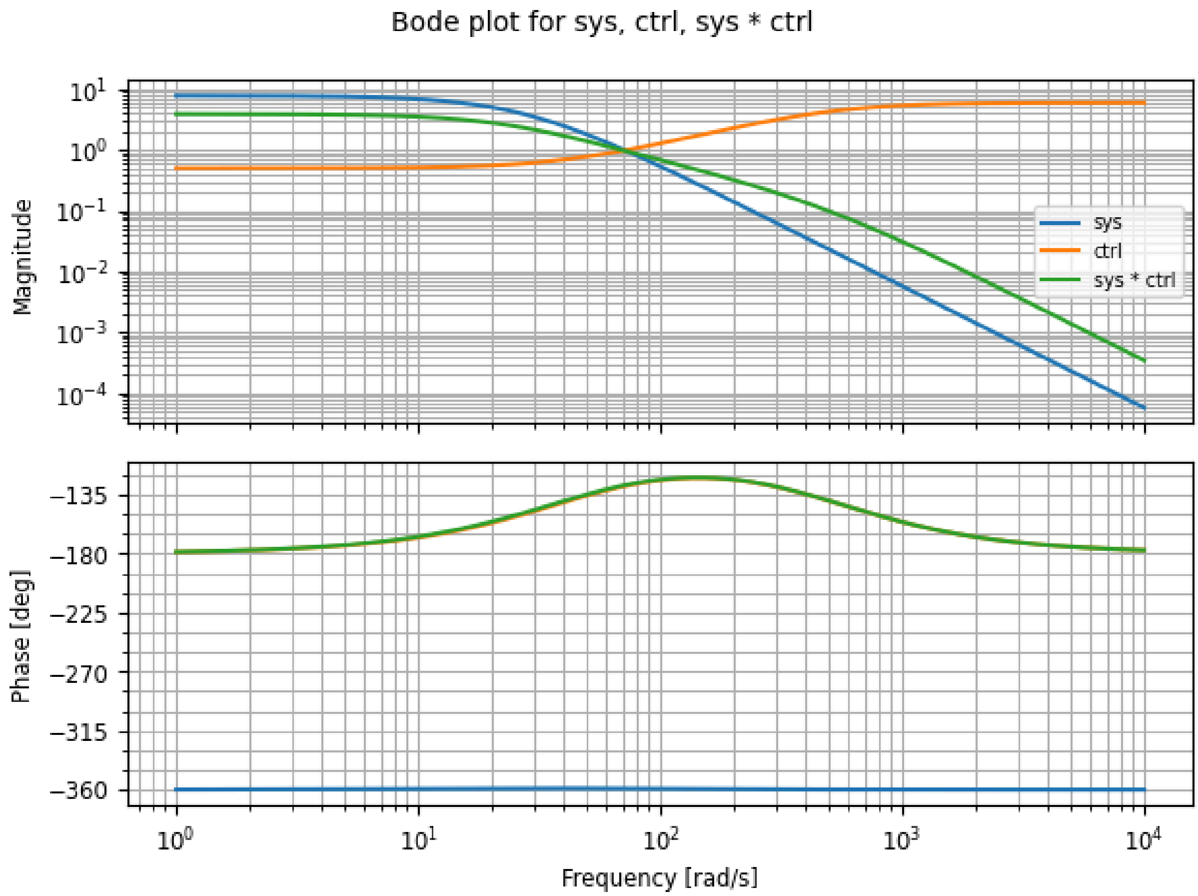
In [6]:
```
# Bode plots
plt.figure(figsize=[5, 3.75])
magC1.name = "ctrl"
out = ct.bode_plot([magP, magL, magC1], np.logspace(0, 4), initial_phase=0)
axs = ct.get_plot_axes(out)
plt.suptitle("")
axs[0, 0].set_title("Bode plot for maglev", fontsize=10)
axs[0, 0].set_ylim(0.06, 1.5e1)
```

Out[6]:
```
(0.06, 15.0)
```




In [7]:
```
# Default Bode plot (with no customization)
ct.bode_plot([magP, magC1, magL])
```

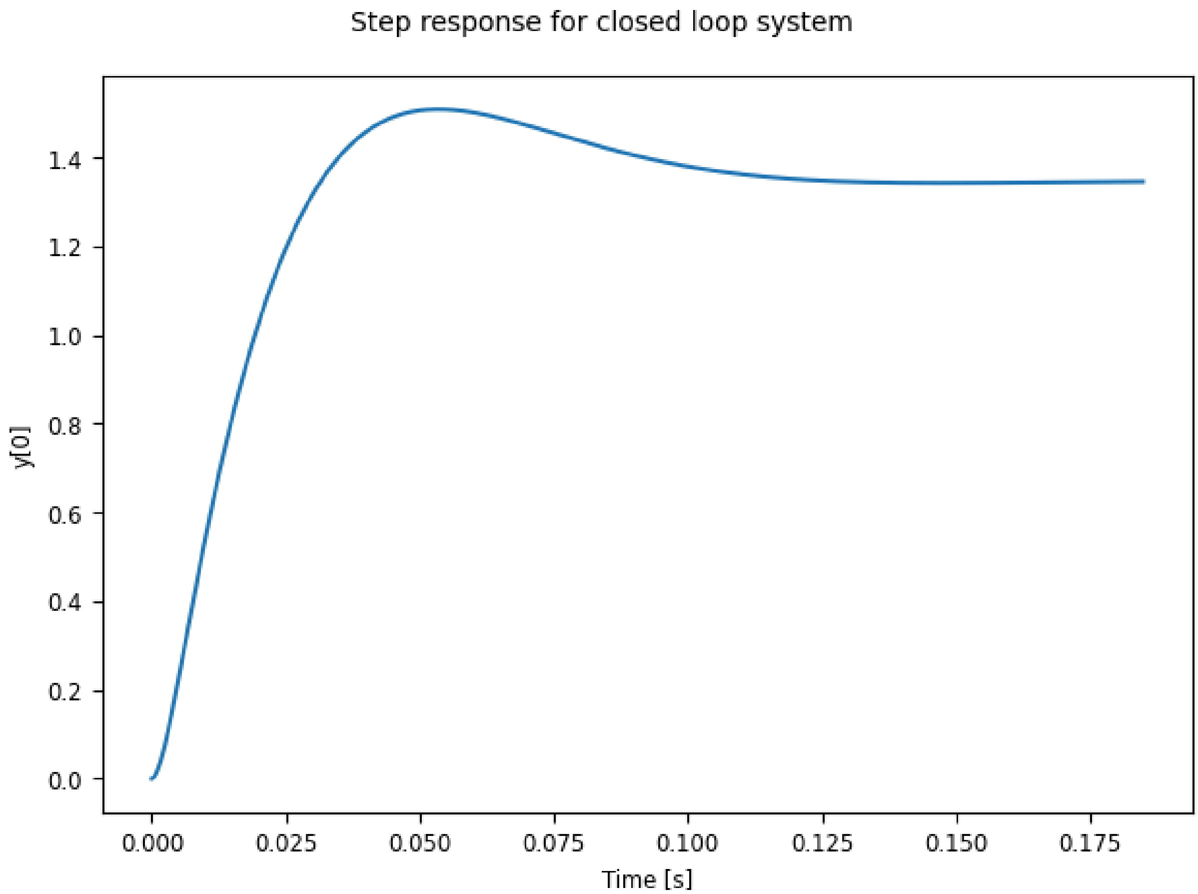
Out[7]:
```
array([[list([<matplotlib.lines.Line2D object at 0x13051c770>, <matplotlib.
lines.Line2D object at 0x13061ce00>, <matplotlib.lines.Line2D object at 0x1
3061d520>])],
       [list([<matplotlib.lines.Line2D object at 0x13061c950>, <matplotlib.
lines.Line2D object at 0x13061d130>, <matplotlib.lines.Line2D object at 0x1
3061d790>])]],
      dtype=object)
```




In [8]:
```python
# Sensitivity function for closed loop system/.
magS1 = ct.feedback(1, magL1)
```

In [9]:
```python
# Step response
magT1 = ct.feedback(magL1)
magT1.name = "closed loop system"
ct.step_response(magT1).plot()
```

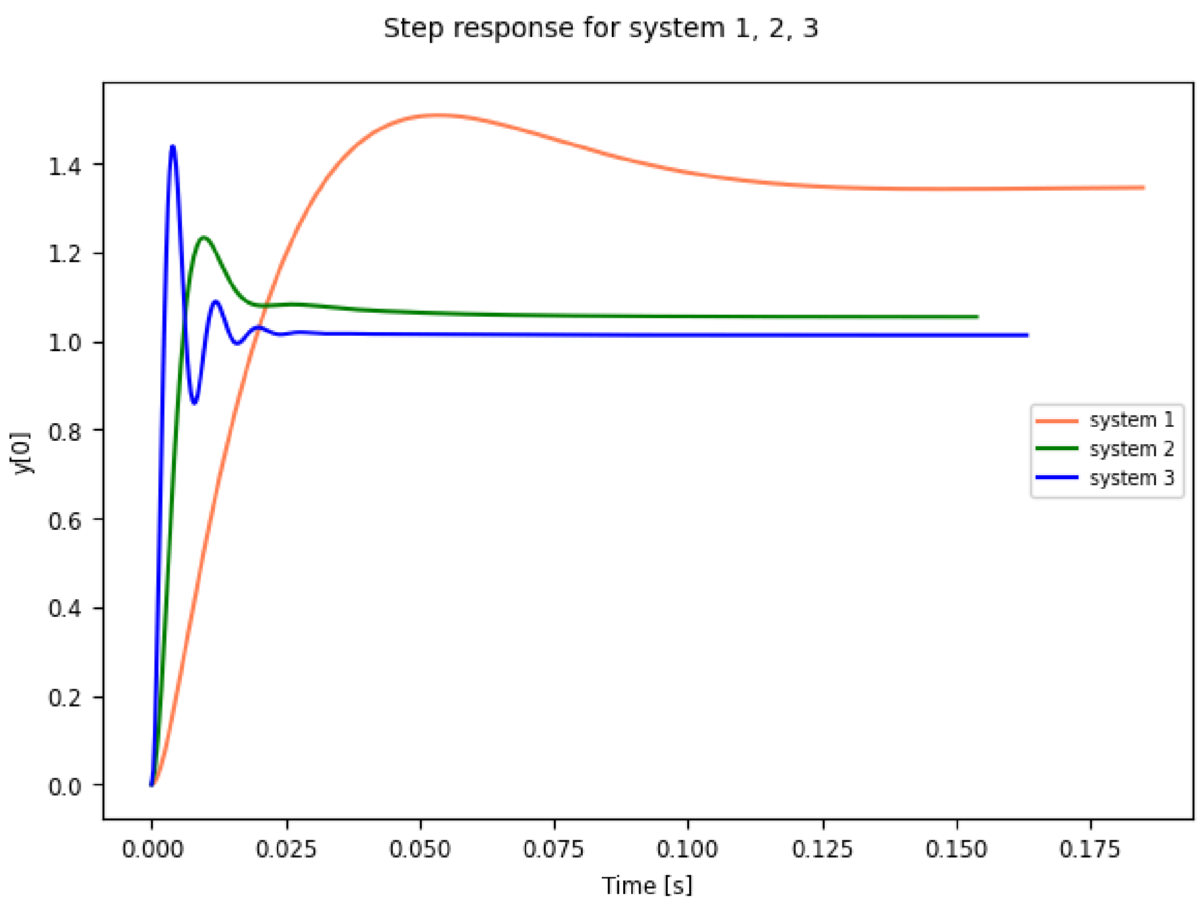
Out[9]:
```
array([[list([<matplotlib.lines.Line2D object at 0x1306f32f0>])]],
      dtype=object)
```

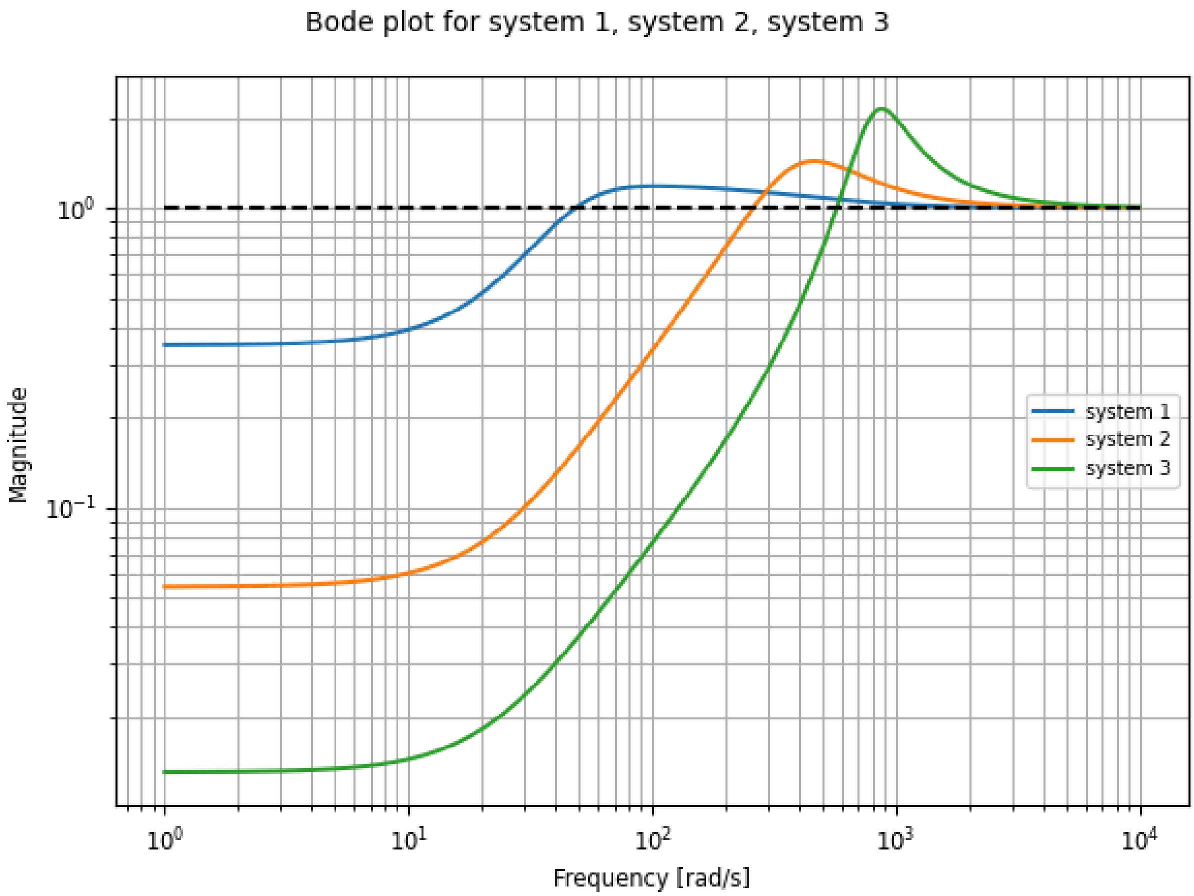



In [10]:
```
# Try to improve performance by increasing DC gain
# System with gain increased
magC2 = magC1*5;                                                # increased gain
magL2 = magP * magC2;                                        # loop transfer func
magS2 = ct.feedback(1, magP * magC2);       # sensitivity function
magT2 = ct.feedback(magP * magC2, 1);       # closed loop response

# System with gain increased even more
magC3 = magC1*20;                                               # increased gain
magL3 = magP*magC3;                                            # loop transfer fu
magS3 = ct.feedback(1, magP * magC3);       # sensitivity function
magT3 = ct.feedback(magP * magC3, 1);       # closed loop response

# name systems
magT1.name = "system 1"
magT2.name = "system 2"
magT3.name = "system 3"
magS1.name = "system 1"
magS2.name = "system 2"
magS3.name = "system 3"

# Plot step responses for different systems
colors = ['b', 'g', '#FF7F50']
for sys in [magT1, magT2, magT3]:
    ct.step_response(sys).plot(color=colors.pop())

# Bode plot for sensitivity function
plt.figure()
```

```
lines = ct.bode_plot([magS1, magS2, magS3], plot_phase=False)

# Add magnitude of 1
xdata = lines[0][0][0].get_xdata()
ydata = np.ones_like(xdata)
plt.plot(xdata, ydata, color='k', linestyle='--')
```

Out[10]: [<matplotlib.lines.Line2D at 0x131241e50>]






In [11]:
```
# Bode integral calculation
omega = np.linspace(0, 1e6, 100000)
for name, sys in zip(['C1', 'C2', 'C3'], [magS1, magS2, magS3]):
    freqresp = ct.frequency_response(sys, omega)
    bodeint = np.trapz(np.log(freqresp.magnitude), omega)
    print("Bode integral for", name, "=", bodeint)

print("pi * sum[ Re(pk) ]", pi * np.sum(magP.poles()[magP.poles().real > 0]))
```

```
Bode integral for C1 = [[84.10451626]]
Bode integral for C2 = [[83.96609718]]
Bode integral for C3 = [[83.44702558]]
pi * sum[ Re(pk) ] (84.13912140726038+0j)
```

In [ ]: