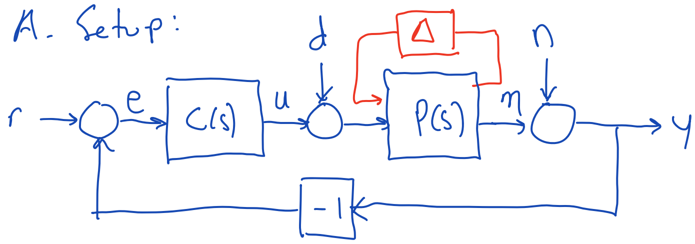
CDS 110, Lecture 8.2: Limits on performance

RMM 21 May 2024

I. Sensitivity functions

A. Setup:



$$\begin{bmatrix} y \\ u \\ e \\ \mu \\ \eta \end{bmatrix} = \begin{bmatrix} \text{sensitivity} \\ \text{fcns} \end{bmatrix} \begin{bmatrix} r \\ d \\ n \end{bmatrix} \begin{matrix} \\ \leftarrow v \\ \leftarrow w \end{matrix}$$

B. Alternative interpretation: process uncertainty Δ

$$G_{yd} = \frac{P}{1+PC} \qquad \frac{dG_{yd}}{dP} = \frac{1}{1+PC} - \frac{PC}{(1+PC)^2} = \frac{1}{(1+PC)^2} = S\,\frac{G_{yd}}{P}$$

$$\Rightarrow \quad \frac{dG_{yd}}{G_{yd}} = S\,\frac{dP}{P} \quad \Rightarrow \text{ relative change in } G_{yd} \text{ is given by "sensitivity"}$$

Similarly $\quad \dfrac{dG_{un}}{G_{un}} = -T\,\dfrac{dP}{P} \Rightarrow$ relative change in G_{un} given by "complementary sensitivity"

$S + T = 1 \Rightarrow$ can't keep both small (at the same frequency)

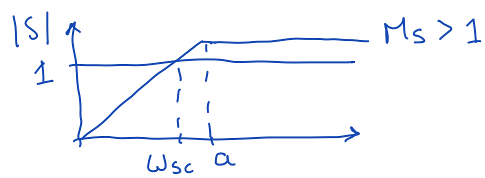
II. Limits of RHP poles and zeros

A. Maximum modulus thm: G analytic in Ω, $\sup\limits_{s\in\Omega} |G(s)| = \sup\limits_{s\in\partial\Omega} |G(s)|$

$$\Rightarrow \quad \max_{\omega\in\mathbb{R}} |G(i\omega)| = \max_{\mathrm{Re}\, s \geq 0} |G(s)|$$

B. Application to RHP poles and zeros

RHP zero



Spec: $|S(i\omega)| \leq |S_r(i\omega)| \Rightarrow \left|\dfrac{S(i\omega)}{S_r(i\omega)}\right| \leq 1$

$$\cancel{S(z)}^{\,1}\,\frac{z+a}{M_s z} \leq \underbrace{\max_{RHP}\left|\frac{S(s)}{S_r(s)}\right|}_{MMP} = \underbrace{\max_{\omega}\left|\frac{S(i\omega)}{S_r(i\omega)}\right| \leq 1}_{spec}$$

$$S_r(s) = \frac{M_s\, s}{s+a}$$

$$\Rightarrow \underbrace{a \leq z(M_s - 1)}_{\text{tradeoff!}} \;\;\&\;\; \omega_{sc} \leq z\sqrt{\frac{M_s-1}{M_s+1}} \leftarrow \text{via some algebra}$$

- Implication: "slow" RHP zeros limit range over which S can be small
- Intuition via step response: have to wait for inverse response
- Similar result for RHP pole (acting on T)
- RHP pole & zero: can show $M_s \geq \left|\dfrac{z+p}{z-p}\right|$
  $\Rightarrow$ nearby pole/zero very hard
- Important insight: zeros depend on sensor (C matrix)

III. Big picture (if time)