CDS 110, Lecture 6.2: trajectory tracking

RMM 6 May 2024

I. Trajectory tracking

A. $\dot{x} = f(x,u)$ Let $e = x - x_d$, $v = u - u_d$

$\dot{x}_d = f(x_d, u_d)$

$$\dot{e} = f(x,u) - f(x_d,u_d) =: F(e,v,x_d,u_d) = f(x_d+e, u_d+e) - f(x_d,u_d)$$

Now stabilize $e=0 \Rightarrow v = -Ke \Rightarrow u = u_d - K(x - x_d)$

B. Choosing gain: Find a "representative" eq pt x_e, u_e and linearize

II. Gain scheduling

- Define $A(x_d,u_d) = \frac{\partial F}{\partial e}\Big|_{0,0} = \frac{\partial f}{\partial x}\Big|_{x_d,u_d}$, $B(x_d,u_d) = \frac{\partial f}{\partial u}\Big|_{x_d,u_d}$

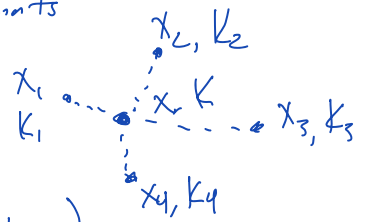
  Locally $\dot{e} \approx A(x_d,u_d)\,e + B(x_d,u_d)\,v$

  Choose $K(x_d,u_d) = \text{lqr}(A(x_d,u_d), B(x_d,u_d), Q_x, Q_u)$
- Problem: need different K at each point on trajectory
- Solution: choose fixed set of points and interpolate

  Let x_i, u_i, $i = 1, \dots, k$ be representative points

  $K_i = \text{lqr}(A(x_i,u_i), \dots)$

  Given current state x, interpolate gains



- Variation: schedule on any variable $\mu = \gamma(x,u)$ (example: vehicle speed)

III. Integral action: can also include $k_i \int h(x) - h(x_d)\, d\tau$

whatever variable you want to drive to zero

IV. Implementation in Python

create_statefbk_iosystem()