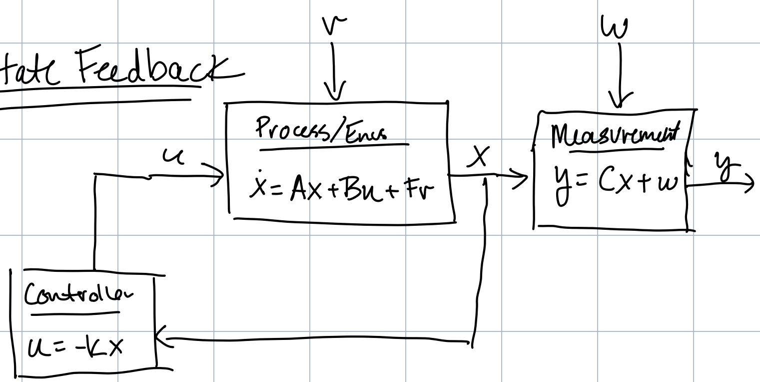
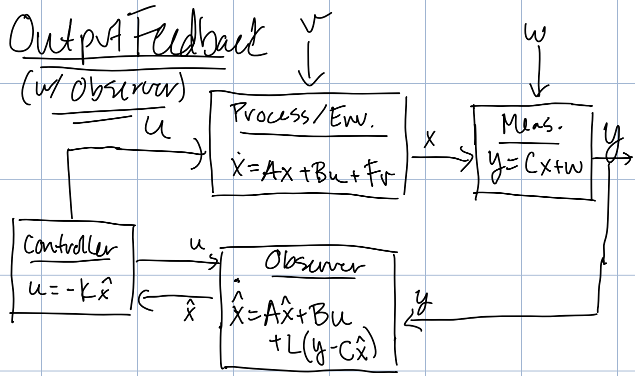
CDS110, Lecture 5.2: Output Feedback

NMB, May 1 2024

State Feedback



Output Feedback
(w/ Observer)



Closed loop dynamics:

Goal: Describe system dynamics as fn. of $x, \check{x}, v, w$ (remove $\hat{x}$ from all eqns)

$$\frac{dx}{dt} = Ax - BK\hat{x} + Fv$$
$$= Ax - BK(x - \check{x}) + Fv \longleftarrow$$
$$= (A - BK)x + BK\check{x} + Fv$$

Let $\underbrace{x = \hat{x}}_{\text{estimate}} + \underbrace{\check{x}}_{\text{error}}$

$$\Rightarrow \check{x} = x - \hat{x}$$

$$\frac{d\check{x}}{dt} = \frac{dx}{dt} - \frac{d\hat{x}}{dt}$$
$$= \left[(A-BK)x + BK\check{x} + Fv\right] \longleftarrow$$
$$- \left[(A-BK)\hat{x} + LC\check{x} + Lw\right]$$
$$= (A-\cancel{BK})\check{x} + \cancel{BK\check{x}} - LC\check{x} + Fv - Lw$$
$$= (A - LC)\check{x} + Fv - Lw$$

$$\frac{d\hat{x}}{dt} = A\hat{x} - BK\hat{x} + L(\overset{Cx+w}{y} - C\hat{x})$$
$$= A\hat{x} - BK\hat{x} + LCx + Lw - LC\hat{x}$$
$$= (A - BK)\hat{x} + LC\check{x} + Lw$$

$$\rightarrow \frac{d}{dt}\underbrace{\begin{bmatrix} x \\ \check{x} \end{bmatrix}}_{\text{state}} = \underbrace{\begin{bmatrix} A - BK & BK \\ 0 & A - LC \end{bmatrix}}_{\text{dynamics}} \begin{bmatrix} x \\ \check{x} \end{bmatrix} + \underbrace{\begin{bmatrix} Fv \\ Fv - Lw \end{bmatrix}}_{\text{inputs}}$$

Separation Principle

For block diagonal, invertible E,

$$\det\begin{bmatrix} E & F \\ G & H \end{bmatrix} = \det(E)\det(H - GE^{-1}F)$$

$$\Rightarrow \det\begin{bmatrix} E & F \\ 0 & H \end{bmatrix} = \det(E)\det(H)$$

$$\lambda(s) = \det\begin{bmatrix} sI - A + BK & BK \\ 0 & sI - A + LC \end{bmatrix}$$

$$= \underbrace{\det(sI - A + BK)}_{\lambda_{A-BK}(s)}\underbrace{\det(sI - A + LC)}_{\lambda_{A-LC}(s)}$$

LQR vs. Kalman Filter

LQR

$K, S, \_ = \text{ct.lqr(sys, Q, R)}$

$K = R^{-1}B^TS$

Soln. to: $A^TS + SA - SBR^{-1}B^TS + Q = 0$

let $\text{cov}(v) = \Sigma_v$, $\text{cov}(w) = \Sigma_w$

KF

$L, P, \_ = \text{ct.lqe(sys, } \Sigma_v, \Sigma_w)$

$L = PC^T\Sigma_w^{-1}$

Soln to: $AP + PA^T - PC^T\Sigma_w^{-1}CP + \Sigma_v = 0$

$$\begin{array}{cccc|cc} A & B & K & S & Q & R \\ \updownarrow & \updownarrow & \updownarrow & \updownarrow & \updownarrow & \updownarrow \\ A^T & C^T & L^T & P & \Sigma_v & \Sigma_w \end{array}$$