CALIFORNIA INSTITUTE OF TECHNOLOGY
Control and Dynamical Systems

CDS 101

R. M. Murray
Fall 2008

Problem Set #8

Issued: 24 Nov 08
Due: 3 Dec 08

Note: In the upper left hand corner of the *second* page of your homework set, please put the number of hours that you spent on this homework set (including reading).

1. Consider a second-order process of the form

$$P(s) = \frac{k}{s^2 + 2\zeta\omega_0 s + \omega_0^2} \qquad k, \zeta, \omega_0 > 0.$$

In this problem we will explore various methods for designing a PID controller for the system.

(a) (Eigenvalue assignment) Suppose that we want the closed loop dynamics of the system to have a characteristic polynomial given by

$$p(s) = s^3 + a_1 s^2 + a_2 s + a_3.$$

Compute a formula for the controller parameters of a PID controller (k_p, k_i and k_d) that gives the desired closed loop response.

(b) (Eigenvalue assignment) Let the process parameters be given by $k = 1$, $\zeta = 0.5$ and $\omega_0 = 2$. Using the formulas from part (a), compute a feedback control law that places the closed loop poles of the system at $\lambda = \{-1, -2 \pm j\}$. Plot the step response and frequency response for the closed loop systems, and compute the gain and phase margins for your design.

(c) (Ziegler–Nichols step response) Using the same process parameters, plot the step response for the corresponding system and use the Ziegler–Nichols rules to design PID controllers that stabilize the system. Plot the step response and frequency response for each of your controller designs, and compute the gain and phase margins for each design.

2. For the control systems below, design a P, PI, PD or PID control law that stabilizes the system, gives less than 10% error at zero frequency and gives at least 30° phase margin. You may use any method (loop shaping, Ziegler–Nichols, eigenvalue assignment, etc) and you only need to design one type of controller (as long as it meets the specification). For the closed loop system, determine that steady-state error in response to a step input and the maximum frequency for which the closed loop system can track with less than 25% error.

(a) Disk drive read head positioning system:

$$P(s) = \frac{1}{s^3 + 10s^2 + 3s + 10}$$

(b) Drug administration/compartment model (AM07, Section 3.6):

$$P(s) = \frac{1.5s + 0.75}{s^2 + 0.7s + 0.05}$$

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CDS 110a

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   (a) Disk drive read head positioning system:

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2. Consider a first-order system with a PI controller given by

$$P(s) = \frac{b}{s+a} \qquad C(s) = k_p\Big(1 + \frac{1}{T_i s}\Big).$$

   In this problem we will explore how varying the gains k_p and T_i affect the closed loop dynamics.

   (a) Suppose we want the closed loop system to have the characteristic polynomial

$$s^2 + 2\zeta\omega_0 s + \omega_0^2.$$

   Derive a formula for k_p and T_i in terms of the parameters a, b, ζ and ω_0.

   (b) Suppose that we choose $a = 1$, $b = 1$ and choose ζ and ω_0 such that the closed loop poles of the system are at $\lambda = \{-20 \pm 10j\}$. Compute the resulting controller parameters k_p and T_i and plot the step and frequency responses for the system.

   (c) Using the process parameters from part (b) and holding T_i fixed, let k_p vary from 0 to ∞ (or something very large). Plot the location of the closed loop poles of the system as the gain varies. You should plot your results in two different ways:

   - A pair of plots showing the real and imaginary parts of the poles as a function of the gain k_p, similar to Figure 4.18a in the text.
   - A parametric plot, showing the location of the eigenvalues on the complex plane, as k_p varies. Label the gains at which any interesting features in this plot occur. (This type of plot is called a *root locus* diagram.)

You may find it convenient to use the `subplot` command in MATLAB so that you can present all of your results in a single figure.

3. In this problem we will design a PID compensator for a vectored thrust aircraft (see Example 2.9 in the text for a description). Use the following transfer function to represent the dynamics from the lateral input to the roll angle of the aircraft:

$$P(s) = \frac{r}{Js^2 + cs + mgl} \qquad \begin{array}{lll} g = 9.8\ \text{m/s}^2 & m = 1.5\ \text{kg} & c = 0.05\ \text{kg/s} \\ l = 0.05\ \text{m} & J = 0.0475\ \text{kg m}^2 & r = 0.25\ \text{m} \end{array}$$

   (these parameters correspond to a laboratory-scale experiment that we have a Caltech). Design a feedback controller that tracks a given reference input with the following specifications:

   - Steady-state error of less than 1%
   - Tracking error of less than 5% from 0 to 1 Hz (remember to convert this to rad/s).
   - Phase margin of at least 30°.

   (a) Plot the open loop Bode plot for the system and mark on the plot the various frequency domain constraints in the above specification, as we did in class.

   (b) Design a compensator for the system that satisfies the specification. You should include appropriate plots or calculations showing that all specifications are met.

   (c) Plot the step and frequency response of the resulting closed loop control. For the step response, compute the steady-state error, rise time, overshoot and settling time of your controller.

   (Hint: you may not need all of the terms in a PID controller.)

4. Åström and Murray, Exercise 10.10. Use $k_t = 1$ and steps of magnitude 1, 1.5 and 3 to demonstrate the effect of the anti-windup compensation.

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CDS 210

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1. Åström and Murray, Exercise 10.6

2. [DFT 6.4, page 103] Let

$$P(s) = 4\frac{s-2}{(s+1)^2}$$

and suppose that C is an internally stabilizing controller such that $\|S\|_\infty = 1.5$. Give a positive lower bound for

$$\max_{0\leq\omega\leq 0.1} |S(j\omega)|.$$

3. [DFT 6.5, page 104] Define $\epsilon = \|W_1 S\|_\infty$ and $\delta = \|CS\|_\infty$, so that ϵ is a measure of tracking performance and δ measures control effort. Show that for every point s_0 with Re $s_0 \geq 0$,

$$|W_1(s_0)| \leq \epsilon + |W_1(s_0)P(s_0)|\,\delta.$$

Hence ϵ and δ cannot both be very small and so we cannot get good tracking without exerting some control effort.

4. [DFT 6.6, page 104] Let ω be a frequency such that $j\omega$ is not a pole of P. Suppose that

$$\epsilon := |S(j\omega)| < 1.$$

Derive a lower bound for $C(j\omega)$ that blows up as $\epsilon \to 0$. Hence good tracking at a particular frequency requires large controller gain at this frequency.

5. [DFT 6.7, page 104] Consider a plant with transfer function

$$P(s) = \frac{1}{s^2 - s + 4}$$

and suppose we want to design an internally stabilizing controller such that

   (a) $|S(j\omega| \leq \epsilon$ for $0 \leq \omega \leq 0.1$
   (b) $|S(j\omega| \leq 2$ for $0.1 \leq \omega \leq 5$
   (c) $|S(j\omega| \leq 1$ for $5 \leq \omega \leq \infty$

Find a (positive) lower bound on the achievable ϵ.