CALIFORNIA INSTITUTE OF TECHNOLOGY
Control and Dynamical Systems

CDS 210

Problem Set #6

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Fall 2008

Issued: 10 Nov 08
Due: 17 Nov 08

Note: In the upper left hand corner of the *second* page of your homework set, please put the number of hours that you spent on this homework set (including reading).

1. Consider a closed loop system with the loop transfer function

$$L(s) = \frac{k}{(s+a)(s^2 + 2\zeta\omega_0 s + \omega_0^2)}.$$

   (a) Assuming that $a \ll \omega_0$ and $\zeta = 1$, sketch the Bode and Nyquist plots for the system, labeling they key features (in terms of k, a and ω_0).

   (b) For each of the following parameter sets, use the Nyquist criterion to determine if the closed loop system is stable and, if so, what the gain, phase and stability margins are:

      i. $k = 200$, $a = 1$, $\zeta = 1$, $\omega_n = 10$
      ii. $k = 100$, $a = 1$, $\zeta = 0.1$, $\omega_n = 10$
      iii. $k = 100$, $a = 0$, $\zeta = 1$, $\omega_n = 10$
      iv. $k = 80$, $a = -1$, $\zeta = 1$, $\omega_n = 10$

   Be sure to show the Nyquist plot for each case and show the gain and phase margins on the Nyquist plots.

2. [DFT 3.1, page 44] Show that for a unity feedback system it suffices to check only two transfer functions to determine internal stability.

3. [DFT 3.2, page 44] Let

$$\widehat{P}(s) = \frac{1}{10s+1} \qquad \widehat{C}(s) = k \qquad \widehat{F}(s) = 1.$$

   Find the least positive gain k such that the following are all true:

   (a) The feedback system is internally stable
   (b) $|e(\infty)| \leq 0.1$ when $r(t)$ is the unit step and $n = d = 0$.
   (c) $\|y\|_\infty \leq 0.1$ for all $d(t)$ such that $\|d\|_2 \leq 1$ when $r = n = 0$.

4. [DFT 3.3, page 44] Consider a unity gain feedback system with $r = n = 0$ and $d(t) = \sin(\omega(t)1(t)$. Prove that if the feedback system is internally stable then $y(t) \to 0$ as $t \to \infty$ if and only if $\widehat{P}$ has a zero at $s = i\omega$ or $\widehat{C}$ has a pole at $s = i\omega$.

5. (ÅM08, Exercise 9.10) Consider a system whose input/output response is modeled by $G(s) = 6(-s+1)/(s^2 + 5s + 6)$, which has a zero in the right half-plane.

   (a) Sketch the Bode plot for the system. (Hint: try sketching these by hand first and use MATLAB only if you get stuck.)

(b) Compute the step response for the system, and show that the output goes in the wrong direction initially, which is also referred to as an *inverse-response*.

(c) Compare the response to a minimum phase system by replacing the zero at $s = 1$ with a zero at $s = -1$. Show that the gain curve on the Bode plot is unchanged, but that the phase curve and step response are (significantly) different.