CALIFORNIA INSTITUTE OF TECHNOLOGY
Control and Dynamical Systems

CDS 101/110
Homework Set #8

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All students should complete the following problems:

1. Consider the dynamics of the magnetic levitation system from lecture. The transfer function from the electromagnet input voltage to the IR sensor output voltage is given by

$$P(s) = \frac{k}{s^2 - r^2}$$

with $k = 4000$ and $r = 25$ (these parameters are slightly different than those used in the MATLAB files distributed with the lecture).

   (a) Design a stabilizing compensator for the process, assuming unity feedback. Compute the poles and zeros for the loop transfer function and for the closed loop transfer function between the reference input and measured output.

   (b) Plot the Nyquist plot corresponding to your compensator and verify that the Nyquist criterion is satisfied.

   (c) Plot the log of the magnitude of the sensitivity function, $\log|S(j\omega)|$, versus ω on a *linear* scale and numerically verify that the Bode integral formula is (approximately) satisfied. (Hint: you can do the integration numerically in MATLAB, using the trapz function. Make sure to choose your frequency range sufficiently large.)

2. Consider a second order system with transfer fuction

$$P(s) = \frac{-s+1}{(s+10)^2}.$$

   (a) Plot the Bode plot for the system. Find another transfer function with the same magnitude but whose phase is less negative than the phase of $P(s)$.

   (b) Plot the step response of the open loop system and show that the response initially moves in the opposite direction of the step.

   (c) Consider a proportional controller $C(s) = K_p$. Compute the range of gains for which the controller stabilizes the system and show that as $K_p \to \infty$, one of the poles of the closed loop transfer function approaches the zero at $s = 1$.

Only CDS 110a students need to complete the following additional problems.

3. In this problem we will design a PID compensator for the pitch axis of the Caltech ducted fan. Use the following transfer function to represent the vehicle dynamics:

$$P(s) = \frac{r}{Js^2 + bs + mgl} \qquad \begin{array}{lll} g = 9.8 \text{ m/sec}^2 & m = 1.5 \text{ kg} & b = 0.05 \text{ kg/sec} \\ l = 0.05 \text{ m} & J = 0.0475 \text{ kg m}^2 & r = 0.25 \text{ m} \end{array}$$

   (a) Design a PID compensator that gives steady state error less than 10% and provides phase margin of at least 30°. You can use any method that you choose to compute the gains and you may set some gains to zero (eg, use PI or PD control) if you like. Plot the pole zero diagram, frequency response, and step response for the *closed loop* system.

   (b) Plot the root locus plot for the system. Mark the open and closed loop pole locations corresponding to the PID compensator at the default gain (from part (a)).

(c) Use the root locus plot to choose a new gain such that the dominant poles have a settling time of half of the value of the settling time for the original compensator designed in part (a). Show the location of the the poles with the new gains on your root locus plot. If it is not possible to decrease the settling time, use the root locus plot to explain why.

(d) Assume that the sensor that measures the pitch angle has dynamics of the form

$$\dot{\theta}_m = \tau(\theta - \theta_m)$$

where θ is the true pitch angle, θ_m is the measured pitch angle and $\tau = 1$ is the sensor time constant. For the controller that you designed in part (a), compute the frequency response and step response when the sensor dynamics are included (i.e., you insert the transfer function $H_{\theta_m \theta}$ in the return path for the system). Is the system still stable with the additional sensor dynamics?