Feedback Systems

An Introduction for Scientists and Engineers

SECOND EDITION

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This is the electronic edition of *Feedback Systems* and is available from http://fbsbook.org. Hardcover editions may be purchased from Princeton University Press, http://press.princeton.edu/titles/8701.html.

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Preface to the Second Edition

The second edition of *Feedback Systems* contains a variety of changes that are based on feedback on the first edition, particularly in its use for introductory courses in control. One of the primary comments from users of the text was that the use of control tools for design purposes occurred only after several chapters of analytical tools, leaving the instructor having to try to convince students that the techniques would soon be useful. In our own teaching, we find that we often use design examples in the first few weeks of the class and use this to motivate the various techniques that follow. This approach has been particularly useful in engineering courses, where students are often eager to apply the tools to examples as part of gaining insight into the methods. We also found that universities that have a laboratory component attached to their controls class need to introduce some basic design techniques early, so that students can be implementing control laws in the laboratory in the early weeks of the course.

To help emphasize this more design-oriented flow, we have added a new chapter on "Feedback Principles" that illustrates some simple design principles and tools that can be used to show students what types of problems can be solved using feedback. This new chapter uses simple models, simulations, and elementary analysis techniques, so that it should be accessible to students from a variety of engineering and scientific backgrounds. For courses in which students have already been exposed to the basic ideas of feedback, perhaps in an earlier discipline-specific course, this new chapter can easily be skipped without any loss of continuity.

We have also rearranged some of the material in the final chapters of the book, moving material on fundamental limits from the chapters on frequency domain design (Chapter 11 in the original text, now Chapter 12) and robust performance (Chapter 12 in the original text, now Chapter 13) into a separate chapter on fundamental limits (Chapter 14). This new chapter also contains some additional material on techniques for robust pole placement as well as on limits imposed by nonlinearities.

For the electronic versions of the text, we have added a new chapter to the end of the book, focused on control architectures and design. Our intention in this chapter is to provide a systems view that describes how control design is integrated into a larger model-based development framework, motivated in part by our consulting activities with large companies. In this new chapter we also take the opportunity to present some overview material on "bottoms up" and "top down" approaches to control architectures, briefly introducing some of the many additional concepts from the field of control that are in widespread use in applications.

In addition to these relatively large changes, we have made many other smaller changes based on the feedback we have received from early adopters of the text. We have added some material on the Routh–Hurwitz criterion and root locus plots, to at least serve as "hooks" for instructors who wish to cover that material in more detail. We have also made some notational changes throughout, most notably changing the symbols for disturbance and noise signals to v and w, respectively. The notation in the biological examples has also been updated to match the notation used in the textbook by Del Vecchio and Murray [DM14].

The electronic version of this text also contains a variety of marginal notes that provide additional information and links to web pages, to enable readers to access supplementary information that may be useful for those interested in more detail. The following symbols in the margin may be used to access supplementary information:

Ⓐ Advanced material with additional details

Ⓕ Frequently asked question; additional details available

Ⓗ Historical information

Ⓛ Link to an external site

Overall, we have tried to maintain the style and organization of the book in a manner that is consistent with our goals for the first edition. In particular, we have targeted the material toward a wide range of audiences rather than any specific discipline. One consequence is that instructors who are teaching department-specific courses may find there are other texts that are better suited to these audiences. Books written over the past few years that are tuned to non-traditional audiences, including Janert [Jan14] (computer science), Del Vecchio and Murray [DM14] (biology), and Bechhoefer [Bec20] (physics). In addition, the textbook *Feedback Control for Everyone* by Albertos and Mareels [AM10] provides a readable introduction requiring minimal mathematical background.

Finally, we are indebted to numerous individuals who have taught out of the text and sent us feedback on changes that would better serve their needs. In addition to the many individuals listed in the preface to the first edition, we would like to thank Kalle Åström, Bo Bernhardsson, Karl Berntorp, Constantine Caramanis, Shuo Han, Björn Olofsson, Noah Olsman, Richard Pates, Jason Rolfe, Clancy Rowley, and André Tits for their feedback, insights, and contributions. Vickie Kearn, our recently-retired editor at Princeton University Press, has continued to serve as an enthusiastic advocate for our efforts and we particularly appreciate her support over the years in our vision for the book and for her advocacy of making the material available for free download.

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Preface to the First Edition

This book provides an introduction to the basic principles and tools for the design and analysis of feedback systems. It is intended to serve a diverse audience of scientists and engineers who are interested in understanding and utilizing feedback in physical, biological, information and social systems. We have attempted to keep the mathematical prerequisites to a minimum while being careful not to sacrifice rigor in the process. We have also attempted to make use of examples from a variety of disciplines, illustrating the generality of many of the tools while at the same time showing how they can be applied in specific application domains.

A major goal of this book is to present a concise and insightful view of the current knowledge in feedback and control systems. The field of control started by teaching everything that was known at the time and, as new knowledge was acquired, additional courses were developed to cover new techniques. A consequence of this evolution is that introductory courses have remained the same for many years, and it is often necessary to take many individual courses in order to obtain a good perspective on the field. In developing this book, we have attempted to condense the current knowledge by emphasizing fundamental concepts. We believe that it is important to understand why feedback is useful, to know the language and basic mathematics of control and to grasp the key paradigms that have been developed over the past half century. It is also important to be able to solve simple feedback problems using back-of-the-envelope techniques, to recognize fundamental limitations and difficult control problems and to have a feel for available design methods.

This book was originally developed for use in an experimental course at Caltech involving students from a wide set of backgrounds. The course was offered to undergraduates at the junior and senior levels in traditional engineering disciplines, as well as first- and second-year graduate students in engineering and science. This latter group included graduate students in biology, computer science and physics. Over the course of several years, the text has been classroom tested at Caltech and at Lund University, and the feedback from many students and colleagues has been incorporated to help improve the readability and accessibility of the material.

Because of its intended audience, this book is organized in a slightly unusual fashion compared to many other books on feedback and control. In particular, we introduce a number of concepts in the text that are normally reserved for second-year courses on control and hence often not available to students who are not control systems majors. This has been done at the expense of certain traditional topics, which we felt that the astute student could learn independently and are often explored through the exercises. Examples of topics that we have included are nonlinear dynamics, Lyapunov stability analysis, the matrix exponential, reachability and observability, and fundamental limits of performance and robustness. Topics that we have de-emphasized include root locus techniques, lead/lag compensation and detailed rules for generating Bode and Nyquist plots by hand.

Several features of the book are designed to facilitate its dual function as a basic engineering text and as an introduction for researchers in natural, information and social sciences. The bulk of the material is intended to be used regardless of the audience and covers the core principles and tools in the analysis and design of feedback systems. Advanced sections, marked by the "dangerous bend" symbol shown here, contain material that requires a slightly more technical background, of the sort that would be expected of senior undergraduates in engineering. A few sections are marked by two dangerous bend symbols and are intended for readers with more specialized backgrounds, identified at the beginning of the section. To limit the length of the text, several standard results and extensions are given in the exercises, with appropriate hints toward their solutions.

To further augment the printed material contained here, a companion web site has been developed:

https://www.fbsbook.org

The web site contains a database of frequently asked questions, supplemental examples and exercises, and lecture material for courses based on this text. The material is organized by chapter and includes a summary of the major points in the text as well as links to external resources. The web site also contains the source code for many examples in the book, as well as utilities to implement the techniques described in the text. Most of the code was originally written using MATLAB M-files but was also tested with LabView MathScript to ensure compatibility with both packages. Many files can also be run using other scripting languages such as Octave, SciLab, SysQuake and Xmath.

The first half of the book focuses almost exclusively on state space control systems. We begin in Chapter 3[*] with a description of modeling of physical, biological and information systems using ordinary differential equations and difference equations. Chapter 4 presents a number of examples in some detail, primarily as a reference for problems that will be used throughout the text. Following this, Chapter 5 looks at the dynamic behavior of models, including definitions of stability and more complicated nonlinear behavior. We provide advanced sections in this chapter on Lyapunov stability analysis because we find that it is useful in a broad array of applications and is frequently a topic that is not introduced until later in one's studies.

The remaining three chapters of the first half of the book focus on linear systems, beginning with a description of input/output behavior in Chapter 6. In Chapter 7, we formally introduce feedback systems by demonstrating how state space control laws can be designed. This is followed in Chapter 8 by material on output feedback and estimators. Chapters 7 and 8 introduce the key concepts of reachability and observability, which give tremendous insight into the choice of actuators and sensors, whether for engineered or natural systems.

[*]Chapter numbers reflect those in the second edition.

The second half of the book presents material that is often considered to be from the field of "classical control." This includes the transfer function, introduced in Chapter 9, which is a fundamental tool for understanding feedback systems. Using transfer functions, one can begin to analyze the stability of feedback systems using frequency domain analysis, including the ability to reason about the closed loop behavior of a system from its open loop characteristics. This is the subject of Chapter 10, which revolves around the Nyquist stability criterion.

In Chapters 11 and 12, we again look at the design problem, focusing first on proportional-integral-derivative (PID) controllers and then on the more general process of loop shaping. PID control is by far the most common design technique in control systems and a useful tool for any student. The chapter on frequency domain design introduces many of the ideas of modern control theory, including the sensitivity function. In Chapter 13, we combine the results from the second half of the book to analyze some of the fundamental trade-offs between robustness and performance. This is also a key chapter illustrating the power of the techniques that have been developed and serving as an introduction for more advanced studies.

The book is designed for use in a 10- to 15-week course in feedback systems that provides many of the key concepts needed in a variety of disciplines. For a 10-week course, Chapters 1–3, 5–7 and 9–12 can each be covered in a week's time, with the omission of some topics from the final chapters. A more leisurely course, spread out over 14–15 weeks, could cover the entire book, with 2 weeks on modeling (Chapters 3 and 2)—particularly for students without much background in ordinary differential equations—and 2 weeks on robust performance (Chapter 13).

The mathematical prerequisites for the book are modest and in keeping with our goal of providing an introduction that serves a broad audience. We assume familiarity with the basic tools of linear algebra, including matrices, vectors and eigenvalues. These are typically covered in a sophomore-level course on the subject, and the textbooks by Apostol [Apo69], Arnold [Arn87] and Strang [Str88] can serve as good references. Similarly, we assume basic knowledge of differential equations, including the concepts of homogeneous and particular solutions for linear ordinary differential equations in one variable. Apostol [Apo69] and Boyce and DiPrima [BD04] cover this material well. Finally, we also make use of complex numbers and functions and, in some of the advanced sections, more detailed concepts in complex variables that are typically covered in a junior-level engineering or physics course in mathematical methods. Apostol [Apo67] or Stewart [Ste02] can be used for the basic material, with Ahlfors [Ahl66], Marsden and Hoffman [MH98], or Saff and Snider [SS02] being good references for the more advanced material. We have chosen not to include appendices summarizing these various topics since there are a number of good books available.

One additional choice that we felt was important was the decision not to rely on a knowledge of Laplace transforms in the book. While their use is by far the most common approach to teaching feedback systems in engineering, many students in the natural and information sciences may lack the necessary mathematical background. Since Laplace transforms are not required in any essential way, we have included them only in an advanced section intended to tie things together for students with that background. Of course, we make tremendous use of *transfer functions*, which we introduce through the notion of response to exponential inputs, an approach we feel is more accessible to a broad array of scientists and engineers.

For classes in which students have already had Laplace transforms, it should be quite natural to build on this background in the appropriate sections of the text.

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