CDS 140: Lecture 1.1
Introduction to Dynamics

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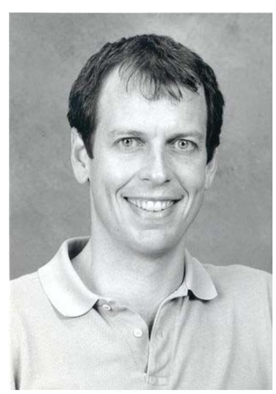
Goals:
- Give an overview of CDS 140: course structure & administration
- Course outline and motivation
- Linear differential equations

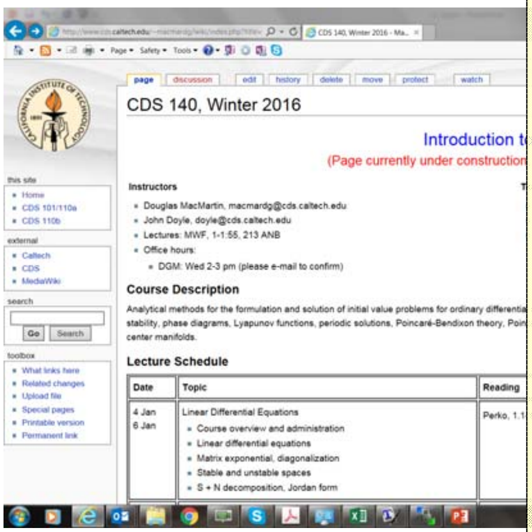
Reading:
- Perko, *Differential Equations & Dynamical Systems, 3rd ed.*, 1.1-1.10

CDS 140
Instructional Staff

- Instructor:
  - Doug MacMartin
  - John Doyle
- TAs (cds140-tas@cds.caltech.edu)
  - Benson Christalin



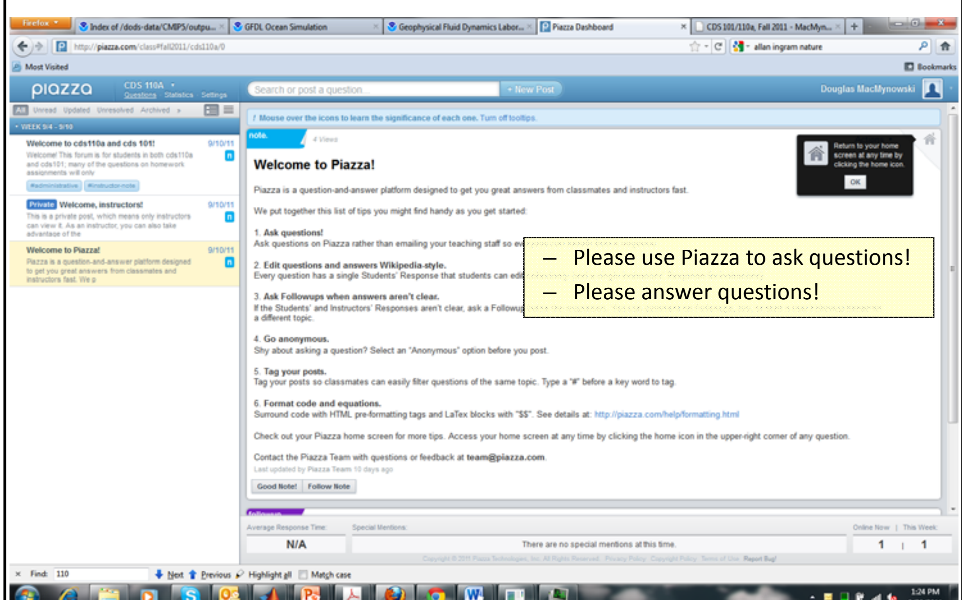
Course Administration



- Class homepage
- Lectures, recitations
  - Week 1, 2 DGM
  - Week 3 JCD
  - Week 4 TBD
- Grading:
  - 75% HW, 25% final
- Homework policy
  - Due Wed 5pm;
  - No extension without prior approval
- Piazza
- Office hours:
  - TBD
- Course text and references
- Course outline

1/4/2016 D. MacMartin, CDS 140 2016 3

Piazza: Website for Questions



- Please use Piazza to ask questions!
- Please answer questions!

Course Outline (tentative)

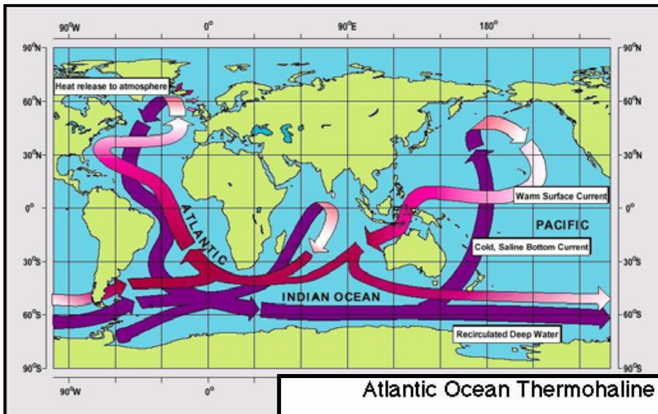
	Week	Topic
1	Jan 4	Linear differential equations (matrix exponential, stable & unstable spaces, Jordan form)
2	Jan 11	Nonlinear differential equations (existence, uniqueness, flow, linearization)
3	Jan 18	Behaviour of differential equations (stable & unstable manifolds)
4	Jan 25	Non-hyperbolic differential equations (center manifold theorem)
5	Feb 4	Global behaviour (limit sets, periodic orbits, limit cycles)
6	Feb 9	Limit cycles (Poincare map)
7	Feb 18	Bifurcations (structural stability, bifurcations of equilibrium points)
8	Feb 25	Bifurcations (Hopf bifurcation, examples)
9	Mar 2	Nonlinear control systems
10	Mar 9	Course review

Overview

- Given some dynamical system:

$$\dot{x} = f(x), \qquad x(0) = x_0$$

- Characterize how the system evolves in time
  - Linear system $\dot{x} = Ax$ is relatively straightforward
    - Given response $x(t)$ to x_0 and $y(t)$ to y_0 then response to x_0+y_0 is $x(t)+y(t)$
  - When does nonlinear system "look" locally like linear?
    - Hyperbolic (linearization has no eigenvalues on imaginary axis)
  - Stable and unstable manifolds, center manifold
  - Limit sets, periodic orbits, limit cycles
  - Bifurcations… how do the dynamics change as parameters change?
    - Many examples… aerodynamic flutter, turbomachinery stall/surge, climate,…

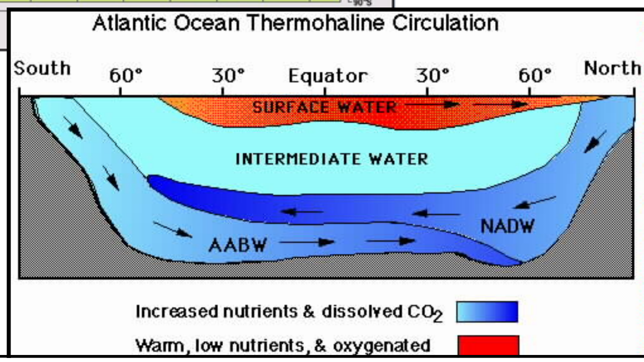


Thermohaline Circulation

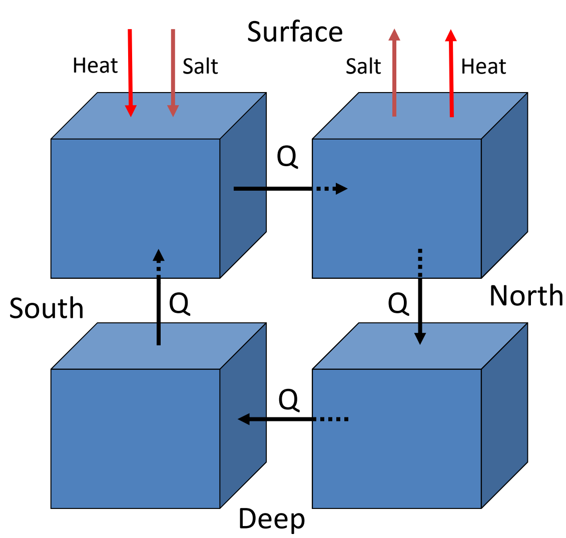
Driven by density gradients (due to temperature and salinity gradients)

- NA: ~20×10^6 m^3/s
- Significant heat transport ~ 1.15PW

For comparison…
US power consump. = 3.23 TW
All rivers into Atlantic = 0.6 Sv
Mississippi river = 0.02 Sv



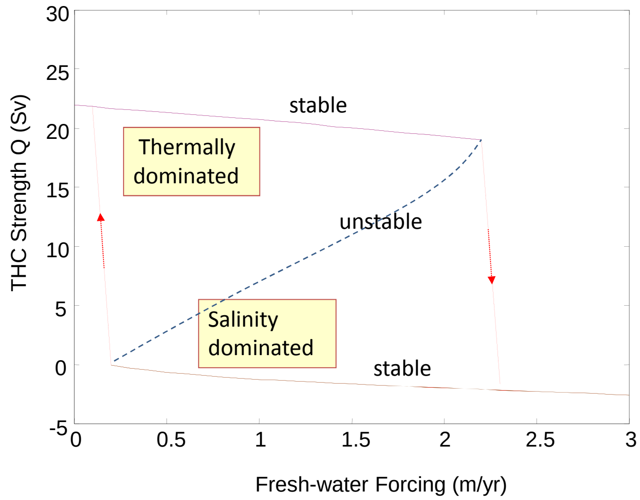
Atlantic Meridional Overturning Circulation (AMOC), THC Dynamics: 4 box model



- After Stommel (1961)
- States are temperature & salinity
- Equatorial evaporation / northern latitude rain results in "freshwater forcing"
  - Bifurcation parameter
- Circulation driven by density gradient

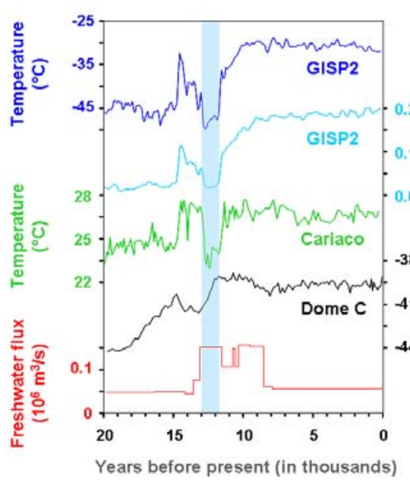
$$\begin{aligned} \rho/\rho_0 &= \alpha T + \beta S \\ Q/Q_0 &= \left(\rho_d^N - \rho_d^S \right. \\ &\quad \left. + \ \delta(\rho_s^N - \rho_s^S)\right) \end{aligned}$$

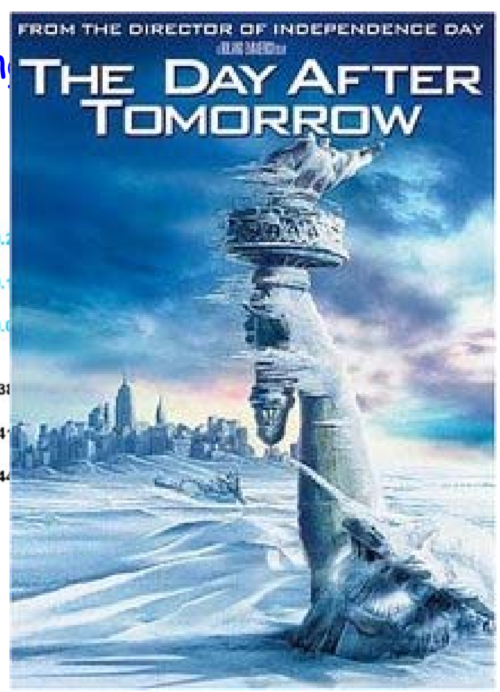
THC Box Model Dynamics



- Both thermal & salinity driven steady solutions exist (as in Stommel 1961)
- Does freshwater melt from Greenland lead to an irreversible (on human time-scale) collapse of MOC?
  - Northern Europe would get colder
- Climate models show weaker circulation with increased CO_2
- Some evidence from paleo record of collapse (younger Dryas)

Youn





Linear Systems

- Nonlinear system $\dot{x} = f(x)$, linearize by taking Taylor series of *f(x)*

$$f(x) \approx f(x_e) + \left.\frac{\partial f}{\partial x}\right|_{x=x_e} (x - x_e) + \cdots$$

- Linear differential equation characterized by $\dot{x} = Ax,\ A \in R^{n\times n}$
  - Assumes equilibrium point shifted to *x=0*
- General solution given by matrix exponential; $x(t) = e^{At}x(0)$
  - Specific case of more general concept: "Flow": $x(t) = \Phi(t, t_0)x(t_0)$
  - Matrix exponential defined by series (need to prove converges)

$$e^A = I + A + \frac{A^2}{2!} + \frac{A^3}{3!} + \ldots$$

- Eigenvalues of *A* determine stability
  - Stable subspace associated with eigenvalues in open LHP
  - Unstable subspace associated with eigenvalues in open RHP
  - Center subspace associated with any eigenvalues with Re(λ)=0
- Need to be careful with non-diagonalizable case, decompose S+N