Feedback Control Theory

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Preface

Striking developments have taken place since 1980 in feedback control theory. The subject has become both more rigorous and more applicable. The rigor is not for its own sake, but rather that even in an engineering discipline rigor can lead to clarity and to methodical solutions to problems. The applicability is a consequence both of new problem formulations and new mathematical solutions to these problems. Moreover, computers and software have changed the way engineering design is done. These developments suggest a fresh presentation of the subject, one that exploits these new developments while emphasizing their connection with classical control.

Control systems are designed so that certain designated signals, such as tracking errors and actuator inputs, do not exceed pre-specified levels. Hindering the achievement of this goal are uncertainty about the plant to be controlled (the mathematical models that we use in representing real physical systems are idealizations) and errors in measuring signals (sensors can measure signals only to a certain accuracy). Despite the seemingly obvious requirement of bringing plant uncertainty explicitly into control problems, it was only in the early 1980s that control researchers re-established the link to the classical work of Bode and others by formulating a tractable mathematical notion of uncertainty in an input-output framework and developing rigorous mathematical techniques to cope with it. This book formulates a precise problem, called the *robust performance problem*, with the goal of achieving specified signal levels in the face of plant uncertainty.

The book is addressed to students in engineering who have had an undergraduate course in signals and systems, including an introduction to frequency-domain methods of analyzing feedback control systems, namely, Bode plots and the Nyquist criterion. A prior course on state-space theory would be advantageous for some optional sections, but is not necessary. To keep the development elementary, the systems are single-input/single-output and linear, operating in continuous time.

Chapters 1 to 7 are intended as the core for a one-semester senior course; they would need supplementing with additional examples. These chapters constitute a basic treatment of feedback design, containing a detailed formulation of the control design problem, the fundamental issue of performance/stability robustness tradeoff, and the graphical design technique of loopshaping, suitable for benign plants (stable, minimum phase). Chapters 8 to 12 are more advanced and are intended for a first graduate course. Chapter 8 is a bridge to the latter half of the book, extending the loopshaping technique and connecting it with notions of optimality. Chapters 9 to 12 treat controller design via optimization. The approach in these latter chapters is mathematical rather than graphical, using elementary tools involving interpolation by analytic functions. This mathematical approach is most useful for multivariable systems, where graphical techniques usually break down. Nevertheless, we believe the setting of single-input/single-output systems is where this new approach should be learned.

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