LPE Reading Group
30 Oct 01

Today:
1) Gradient for Optim. Control Problems
2) Shooting (Fwr, Bck, Multiple)
3) RIOTS
4) Next 2 Lectures
\[ \begin{align*}
\min_x L(x) & \quad \text{s.t. } f(x) = 0 \\
\frac{\partial L}{\partial f} & = -\lambda\\
\frac{\partial}{\partial x_j} \frac{\partial L}{\partial x_i} & \quad (i, j = 1, \ldots, n)\\
\lambda & = -[x^{-1}L_x]^T\\
J^* & = \min_{u(t)} \left\{ \int_0^T L(x(t), u(t), t) dt + V(x(T)) \right\} \\
S^* & = \left\{ \int_0^T \begin{bmatrix} x(t) \\ u(t) \end{bmatrix} dt : x(0) = x_0, u(t) \in U, \right\}
\end{align*} \]
\textbf{P. T) Correlation:} 

- Compute \( x(t) \) via \( x = f(x(t), u(t)) \) \( x(t_0) = x_0 \) given.

- Compute \( \lambda(t) \) \( \dot{\lambda} = -f^T \lambda - L_x \rightarrow \lambda(t) = \frac{2\delta t}{\delta x(t_p)} \).

- Compute \( H_u = (L_u + \lambda^T f_u)(t) \).

- Set \( \delta u = -\varepsilon H_u \).

- Repeat until \( \delta J = -\varepsilon \int_{t_p}^{t_f} (H_u^T H_u) dt \) small.

\[ h = 0 \]
RIOTS  Recursive Integration Optimal Trajectory Solver

Problem

\[
\begin{align*}
\min \{ & \phi(x(t_f), t_f) + \int_{t_0}^{t_f} L(x, u, t) dt \\
\text{s.t.} & x = f(x, u, t), \ x(t_0) \text{ given } \lambda(t) \\
& C(x, u, t) \leq 0, \ t \in [t_0, t_f] \\
& \psi_1(x(t_f)) \leq 0 \\
& \psi_2(x(t_f)) = 0
\end{align*}
\]

Issue with "\leq" is to introduce combinatorial problem

Sketch:

- Gradient algorithm
- Computes \( \psi_i(\cdot) \)
- Gradient by the Adjoint
- \( \frac{\partial f}{\partial x} \psi_1 = \int_{t_0}^{t_f} H u dt \)
- "U" parameter as
- RK ode solver Spline
Splines - piecewise polynomials

NLP solver - NPSOL
   Comm. pkg. (Grill, Murray, Saunders)
   uses SQP
   Major iteration
     guess \( C_{\text{coeff}}^{(0)} \)

\[
J(C_{\text{coeff}}^{(0)}) = J(C_{\text{coeff}}) \]
\[
\frac{\partial}{\partial C_{\text{coeff}}} J(C_{\text{coeff}}) = \frac{\partial}{\partial C_{\text{coeff}}} J_{\text{coeff}} \]
\[
C_{\text{coeff}}^{(n+1)} = C_{\text{coeff}}^{(n)} + \frac{\partial}{\partial C_{\text{coeff}}} J_{\text{coeff}} \frac{\partial}{\partial C_{\text{coeff}}} J_{\text{coeff}}^{-1} \frac{\partial}{\partial C_{\text{coeff}}} J(C_{\text{coeff}}) \]

Outside RIOTS
   everything \((x,t)\)
   determined by \(u\)

\[
\begin{align*}
\text{min } & J(w) \\
\text{st. } & \tilde{c}(w) \leq 0 \\
& (x + 2) \leq 0 \text{ NLP} \\
& -(x + 2) \leq 0 \\
\end{align*}
\]
\[
C_{\text{coeff}} \Rightarrow \tilde{s} \text{ Spline}
\]
Nonmajor Iter's

Quadratic Approx $+ J$
Linear $\rightarrow C$
$\rightarrow$ find $\delta_{\text{coef}}$
Convex Problem

Minor Iterations

Solve

$$\min \left\{ J(c^{\text{coef}}) + DF \delta_{\text{coef}} + \delta_{\text{coef}}^T H \delta_{\text{coef}} \right\}$$

s.t.

$$J(c^{\text{coef}}) + DC \delta_{\text{coef}} \leq 0$$

$\delta_{\text{coef}}$

iterate

$C^{\text{coef}} \leftarrow C^{\text{coef}} + \delta C$

iterate until $J$ small
Nonlinear Trajectory Generation $\mathcal{Z}$

uses collocation: represent $(x, y)$ as $B$-splines: piecewise polynomials refined over intervals (time) by knotpoints.

Look for soln in subspace of $\mathcal{Z}$ (coefficients)

$\begin{align*}
  \dot{x} &= \cos \theta, u,
  x &= \sum c_i \psi_i(t),
  \dot{x} &= \cos \theta \rightarrow \theta, u, \\
  y &= \sin \theta, u,
  \phi &= \tan \theta, u,
  \dot{\phi} &= \dot{u},
\end{align*}$