Goals:
• Show how to use "loop shaping" using PID (Proportional + Integral + Derivative) to achieve a performance specification

Reading:
• Åström and Murray, Feedback Systems, Ch 10
• Advanced: Lewis, Chapters 12-13

Overview of Loop Shaping

Performance specification
- Steady state error
- Tracking error
- Bandwidth
- Relative stability

Approach: “shape” loop transfer function using $C(s)$
• $P(s)$ + specifications given
• $L(s) = P(s) C(s)$
  - Use $C(s)$ to choose desired shape for $L(s)$
• Important: can’t set gain and phase independently
Overview: PID control

\[ u = k_p e + k_i \int e \, dt + k_d \dot{e} \]

Intuition
- Proportional term: provides inputs that correct for "current" errors
- Integral term: ensures steady state error goes to zero
- Derivative term: provides “anticipation” of upcoming changes

A bit of history on “three term control”
- First appeared in 1922 paper by Minorsky: “Directional stability of automatically steered bodies” under the name “three term control”
- Also realized that “small deviations” (linearization) could be used to understand the (nonlinear) system dynamics under control

Utility of PID
- PID control is most common feedback structure in engineering systems
- For many systems, only need PI or PD (special case)
- Many tools for tuning PID loops and designing gains

Proportional Feedback

Simplest controller choice: \( u = k_p e \)
- Effect: lifts gain with no change in phase
- Good for plants with low phase up to desired bandwidth
- Bode: shift gain up by factor of \( k_p \)
- Step response: better steady state error, but with decreasing stability

\[ k_p > 0 \text{ if } P(0) > 0 \]
Proportional + Integral Compensation

Use to eliminate steady state error

- Effect: lifts gain at low frequency
- Gives zero steady state error
- Bode: infinite SS gain + phase lag
- Step response: zero steady state error, with smaller settling time, but more overshoot

\[ P(s) + \frac{k_i}{s} \]

\[ k_p > 0, \ k_i > 0 \]

Proportional + Integral + Derivative (PID)

\[ C(s) = k_p + \frac{k_i}{s} + k_d s \]

\[ = k \left( 1 + \frac{1}{T_i s} + T_d s \right) \]

\[ = \frac{(kT_d)(s + \alpha_i)(s + \alpha_d)}{s} \]

Bode Diagrams
Implementing Derivative Action

Problems with derivatives
• High frequency noise amplified by derivative term
• Step inputs in reference can cause large inputs
• Shows up in Gang of Four...

Solution: modified PID control
• Use high frequency rolloff in derivative term
  - first order filter will give finite gain at high frequency
  - use higher order filter if needed
• Don’t feed reference signal through derivative block
  - Useful when reference has unwanted high frequency content
  - Better solution: reference shaping via two DOF design ($F(s)$ block)
• Many other variations (see AM08 + refs)

Example: Cruise Control using PID - Specification

Performance Specification
• $\leq 1\%$ steady state error
  - Zero frequency gain $> 100$
• $\leq 10\%$ tracking error up to 1 rad/sec
  - Gain $> 10$ from 0-1 rad/sec
• $\geq 45\degree$ phase margin
  - Gives good relative stability
  - Provides robustness to uncertainty

Observations
• Purely proportional gain won’t work: to get gain above desired level will not leave adequate phase margin
• Need to increase the phase from $\sim 0.5$ to 2 rad/sec and increase gain as well
Example: Cruise Control using PID - Design

Approach
- Use integral gain to make steady state error small (zero, in fact)
- Use derivative action to increase phase lead in the cross over region
- Use proportional action to give desired bandwidth

Controller
- $Ti = 1/0.1; Td = 1/1; k = 2000$

$$C(s) = \frac{2000s^2 + 1.1s + 0.1}{s}$$

$$= \frac{2000 + \frac{200}{s} + 2000}{s}$$

Closed loop system
- Very high steady state gain
- Adequate tracking @ 1 rad/sec
- ~80° phase margin
- Verify with Nyquist + Gang of 4

Example: Cruise Control using PID - Verification

Observations
- Very fast response (probably too aggressive)
- Back off on Ti to get something more reasonable
**PID Tuning**

Zeigler-Nichols step response method
- Design PID gains based on step response
- Measure maximum slope + intercept
- Works OK for many plants (but underdamped)
- Good way to get a first cut controller

Zeigler-Nichols frequency response method
- Increase gain until system goes unstable
- Use critical gain and frequency as parameters

Variations
- Modified formulas (see text) give better response
- Relay feedback: provides automated way to obtain critical gain, frequency

### Example: PID cruise control

**Zeigler-Nichols design for cruise controller**
- Plot step response, extract \( \tau \) and \( a \), compute gains

\[
P(s) = \frac{1/m}{s + b/m} \times \frac{r}{s + a}
\]

\( \tau = 2.49 \)

\( a = 0.039 \)

- Result: sluggish \( \Rightarrow \) increase loop gain + more phase margin (shift zero)
Windup and Anti-Windup Compensation

Problem
- Limited magnitude input (saturation)
- Integrator "winds up" ⇒ overshoot

Solution
- Compare commanded input to actual
- Subtract off difference from integrator

Summary: Frequency Domain Design using PID

Loop Shaping for Stability & Performance
- Steady state error, bandwidth, tracking

\[ H_{uc}(s) = K_P + K_I \frac{1}{s} + K_D s \]

Main ideas
- Performance specs give bounds on loop transfer function
- Use controller to shape response
- Gain/phase relationships constrain design approach
- Standard compensators: proportional, PI, PID