Invariant Manifolds, 3-Body Problem & Petit Grand Tour of Jovian Moons

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Acknowledgements

- H. Poincaré, J. Moser
- C. Conley, R. McGehee, D. Appleyard
- C. Simó, J. Llibre, R. Martinez
- B. Farquhar, D. Dunham
- E. Belbruno, B. Marsden, J. Miller
- K. Howell, B. Barden, R. Wilson
Outline of Presentation

Main Theme

- how to use invariant manifold of 3-body problem in space mission design.

Background and Motivation

Major Results and Some Technical Details.

Low Energy Transfer between Jovian Moons.

Conclusion and Ongoing Work.
Jupiter Comets

- **Rapid transition** from outside to inside Jupiter’s orbit.
- **Captured temporarily** by Jupiter during transition.
- **Exterior** (2:3 resonance). **Interior** (3:2 resonance).
Jupiter Comets

- Belbruno and B. Marsden [1997]
- Lo and Ross [1997]
  - Comet in rotat**ing** frame follows invariant manifolds.
- Moser, Conley and McGehee. LLibre, Martinez and Simó [1985].
Genesis Discovery Mission

To understand better heteroclinic dynamics used by Genesis Mission in returning solar wind sample to Earth.

- Howell, Lo, Barden and Wilson.

Halo orbit, transfer/return trajectories in rotating frame.
The Flow near $L_1$ and $L_2$

- For energy value just above that of $L_2$, Hill’s region contains a “neck” about $L_1$ & $L_2$.
- Comet can make transition through these equilibrium regions.
- 4 types of orbits: periodic, asymptotic, transit & nontransit.
Major Result (A): Heteroclinic Connection

- Found **heteroclinic connection** between pair of periodic orbits.
- Found a large class of **orbits** near this (homo/heteroclinic) **chain**.
- Comet can follow these **channels** in rapid transition.
Major Result (B): Existence of Transitional Orbits

- **Symbolic sequence** used to label itinerary of each comet orbit.

- **Main Theorem:** For any admissible itinerary, e.g., (... X, J; S, J, X, ...), there exists an orbit whose whereabouts matches this itinerary.

- Can even specify **number of revolutions** the comet makes around Sun & Jupiter (plus $L_1$ & $L_2$).
■ **Major Result (C): Numerical Construction of Orbits**

- Developed procedure to construct orbit with **prescribed itinerary**.
- Example: An orbit with itinerary \((X, J; S, J, X)\).
Details: Construction of \((J, X; J, S, J)\) Orbits

- Invariant manifold **tubes** separate transit from nontransit orbits.
- **Green curve** (Poincaré cut of \(L_1\) stable manifold).
  **Red curve** (cut of \(L_2\) unstable manifold).
- Any point inside the intersection region \(\Delta_J\) is a \((X; J, S)\) orbit.
Details: Construction of \((J, X; J, S, J)\) Orbits

The desired orbit can be constructed by

- Choosing appropriate Poincaré sections and
- linking invariant manifold tubes in right order.
Petit Grand Tour of Jupiter’s Moons (Planar Model)

- Used invariant manifolds to construct trajectories with interesting characteristics:
  - Petit Grand Tour of Jupiter’s moons. 1 orbit around Ganymede. 4 orbits around Europa.
  - A $\Delta V$ nudges the SC from Jupiter-Ganymede system to Jupiter-Europa system.
- Instead of flybys, can orbit several moons for any duration.
Extend from Planar Model to Spatial Model

- Previous work based on **planar** 3-body problem.
- Future missions will require **3D** capabilities.
  - Europa Orbiter mission needs a capture into a **high inclination orbit** around Europa.
- Current study has extended from **planar** to **spatial model**.
Extend from Planar Model to Spatial Model

- Ganymede's orbit
- Europa's orbit

(a) Close approach to Ganymede
(b) Injection into high inclination orbit around Europa

Image (a): Jupiter, Ganymede's orbit, Europa's orbit
Image (b): Close approach to Ganymede
Image (c): Injection into high inclination orbit around Europa
Petit Grand Tour of Jupiter’s Moons

- Jupiter-Ganymede-Europa-SC 4-body system approximated as 2 coupled 3-body systems
- Invariant manifold tubes of spatial 3-body systems are linked in right order to construct orbit with desired itinerary.
- Initial solution refined in 4-body model.
Planar Restricted 3-Body Problem

- **Recall:** for energy value just above that of \( L_2 \), Hill’s region contains a “neck” about \( L_1 \) & \( L_2 \).

- Dynamics in each equilibrium region: saddle x center.

- 4 types of orbits: periodic, asymptotic, transit & nontransit.
Planar: Invariant Manifold as Separatrix

Asymptotic orbits form **2D invariant manifold tubes** in **3D energy surface**.

They separate transit and non-transit orbits:

- **Transit orbits** are those inside the tubes.
- **Non-transit orbits** are those outside the tubes.
Spatial Restricted 3-Body Problem (CR3BP)

- Dynamics near equilibrium point: saddle x center x center.
  - bounded orbits (periodic/quasi-periodic): $S^3$ (3-sphere)
  - asymptotic orbits to 3-sphere: $S^3 \times I$ (“tubes”)
  - transit and nontransit orbits.
Spatial: Invariant Manifold as Separatrix

Asymptotic orbits form 4D invariant manifold “tubes” \((S^3 \times I)\) in 5D energy surface.

They separate transit and non-transit orbits:

- **Transit orbits** are those inside the “tubes”.
- **Non-transit orbits** are those outside the “tubes”.

![Diagram showing spatial invariant manifold as separatrices](image)
Construct Orbit with Desired Itinerary

▶ How to **link** invariant manifold tubes to construct orbit with **desired itinerary**.

▶ Construction of \((X; M, I)\) orbit.
Planer: Construction of \((X; M, I)\) Orbits

- Invariant mfd. tubes \((S \times I)\) separate transit/nontransit orbits.
- Red curve \((S^1)\) (Poincaré cut of \(L_2\) unstable manifold).
- Green curve \((S^1)\) (cut of \(L_1\) stable manifold).
- Any point inside the intersection region \(\Delta_M\) is a \((X; M, I)\) orbit.
Spatial: Construction of $(X; M, I)$ orbits

- Invariant manifold **tubes**: $(S^3 \times I)$.
- **Poincaré cut** is a topological **3-sphere** $S^3$ in $\mathbb{R}^4$.
  - $S^3$ looks like **disk x disk**: $\xi^2 + \dot{\xi}^2 + \eta^2 + \dot{\eta}^2 = r^2 = r_\xi^2 + r_\eta^2$
- If $z = c$, $\dot{z} = 0$, its **projection** on $(y, \dot{y})$ **plane** is a **curve**.
- Any point inside this **curve** is a $(X; M)$ orbit.
Spatial: Construction of \((X; M, I)\) orbits

- Similarly, while cut of stable manifold “tube” is a \(S^3\), its projection on \((y, \dot{y})\) plane is a curve for \(z = c, \dot{z} = 0\).
- Any point inside this curve is a \((M, I)\) orbit.
- Hence, any point inside the intersection region \(\Delta_M\) is a \((X; M, I)\) orbit.
Spatial: Construction of $(X; M, I)$ orbits
Petit Grand Tour of Jupiter’s Moon

Petit Grand Tour can be constructed similarly

- Approximate 4-body system as 2 nested 3-body systems.
- Choose appropriate Poincaré section.
- Link invariant manifold tubes in right order.
- Refine initial solution in 4-body model.
Conclusion and Future Work

In our study of spatial CR3BP:

- Invariant manifolds still act as separtrices.
- Construct orbit with prescribed itinerary.
- Construct Petit Grand Tour of Jupiter’s moons that ends in a high inclination orbit around Europa.

Will construct useful trajectories in Sun-Earth-Moon system.
Future Work: Pump Down via Resonance Encounters

For this new tour: $\Delta V = 20m/s$. 

Low Energy Tour of Jupiter’s Moons

Seen in Jovicentric Inertial Frame

Callisto
Ganymede
Europa
Jupiter
Koon, W.S., M.W. Lo, J.E. Marsden and S.D. Ross
Heteroclinic connections between periodic orbits and
resonance transitions in celestial mechanics,

http://www.cds.caltech.edu/~koon/
http://ojps.aip.org/chaos/

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Masdemont and S.D. Ross
Invariant Manifolds, The Spatial Three-Body Problem
and Space Mission Design.

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