Controlling Structured Spatially Interconnected Systems

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“Recent” Developments...

• Proliferation of actuators and sensors
• “Moore’s Law”
• Embedded systems, CAN, Bluetooth

MORE PERFORMANCE!!!
What is, and will be, needed:

NEW CONTROL TOOLS:

• LARGE numbers of actuators and sensors
• Distributed computation
• Limited connectivity

• Robustness
• Performance
• Flexibility
• Etc.
Modeling Interconnected Systems

\[ \dot{x}(t) = f(x(t), d(t)) \]
\[ z(t) = h(x(t), d(t)) \]

\[ x(t), d(t), \text{and } z(t) \text{ live in a Hilbert space:} \]

\[ d(t) = d(t, s_1, s_2, \ldots, s_L) = d(t, s) \]
\[ \|d(t)\|_{L_2}^2 := \sum_{s_1 = -\infty}^{\infty} \cdots \sum_{s_L = -\infty}^{\infty} d^*(t, s)d(t, s) \]
Restrict the model class: local interconnection

WHY?

• Large class of systems, non-trivial behavior:
  • vehicle platoons
  • finite difference approximations of PDEs
  • cellular automata, artificial life, etc.
  • behavior of groups, swarm intelligence, etc.
Case Study: Formation Flight

Use upwash created by neighbouring craft to provide extra lift

**MOTIVATION**

- “satellite” type of applications
  (Wolfe, Chichka and Speyer ‘96)
- MAVs and UAVs, extend range
EFFICIENCY OF FORMATION, ELLIPTICAL DISTRIBUTION

Lissaman and Shollenberger '70: Formation Flight of Birds
\[ \ddot{y}(t, s) = c_1 y(t, s) - c_2 \dot{\theta}(t, s) \]

\[ \ddot{\theta}(t, s) = -c_3 \dot{\theta}(t, s) + u(t, s) - c_4 u(t, s + 1) + c_5 \dot{\theta}(t, s + 1) + c_6 \left( y(t, s) - y(t, s + 1) \right)^2 \]

Student:
Jeff Fowler (ME)
Define shift operator $S$:

$$(Su)(t,s) := u(t,s+1)$$

yields

$$\begin{align*}
\ddot{y} &= c_1 y - c_2 \theta \\
\ddot{\theta} &= -c_3 \dot{\theta} + u - c_4 Su + c_5 S \dot{\theta} + c_6 (y - Sy)^2
\end{align*}$$

In general:

$$\Delta = \begin{bmatrix}
\frac{d}{dt} & I \\
S_i & \vdots \\
S_{i-1}^\dagger & \ddots \\
S_{1}^\dagger & I
\end{bmatrix}$$

$$(\Delta x)(t,s) = F(x(t,s), d(t,s), s)$$

$$(\Delta z)(t,s) = H(x(t,s), d(t,s), s)$$

d(t,s), x(t,s), z(t,s) are FD, F and H NL functions on FD space
Special cases...

\[
\Delta x(t, s) = F(x(t, s), d(t, s), s)
\]

\[
z(t, s) = H(x(t, s), d(t, s), s)
\]

\[
\Delta = \frac{d}{dt} I,
\]

\[
F(\cdot) = Ax + Bd, H(\cdot) = Cx + Dd 
\]

\[
\Rightarrow \dot{x} = Ax + Bd, z = Cx + Dd
\]

\[
F(\cdot) = Ax + Bd, H(\cdot) = Cx + Dd
\]

Linear, spatial invariant systems

\[
\Delta = \frac{d}{dt} I
\]

Family of completely decentralized systems

“CONVENIENT” FRAMEWORK FOR CAPTURING STRUCTURE
Recent Related Work

Siljak et al: Decentralized control of complex systems

Bamieh, Paganini, Dahleh: Spatially Invariant Systems

Cheng, Yang, Zhai, Peterson, Savkin, …: Decentralized Control of IC systems.

Stewart, Gorinevski, Dumont: Cross directional control
Control Design and Analysis: Spatially Invariant Systems

\[ \Delta x = Ax + Bd \]
\[ z = Cx + Dd \]

Stable: \((\Delta - A)^{-1}\) exists and is bounded

Contractive: \(\|D + C(\Delta - A)^{-1}B\| < 1\)
**Analysis**

\[
\begin{align*}
\dot{x}_T &= A_{TT} x_T + A_{TS} x_S + B_T d \\
\Delta x_S &= A_{ST} x_T + A_{SS} x_S + B_S d \\
z &= C_T x_T + C_S x_S + D d
\end{align*}
\]

\[
\Delta_s = \begin{bmatrix} s_{1I} & \cdots & s_{L_I} \end{bmatrix}, \quad X_s = \begin{bmatrix} x_1^+ & x_1^- & \cdots & x_L^+ & x_L^- \end{bmatrix}
\]

Stable and Contractive if there exists \( X_T > 0 \) and structured \( X_S \) s.t.

\[
\begin{bmatrix} I & 0 & 0 \\ A_{ST}^- & A_{SS}^- & B_S^- \\ 0 & 0 & I \end{bmatrix}^* \begin{bmatrix} A_{TT}^* X_T + X_T A_{TT} & X_T A_{TS}^+ & X_T B_T \\ \left( X_T A_{TS}^+ \right)^* & -X_S & 0 \\ \left( X_T B_T \right)^* & 0 & -I \end{bmatrix} \begin{bmatrix} I & 0 & 0 \\ A_{ST}^- & A_{SS}^- & B_S^- \\ 0 & 0 & I \end{bmatrix} + \begin{bmatrix} I & 0 & 0 \\ A_{ST}^+ & A_{SS}^+ & B_S^+ \\ C_T & C_S & D \end{bmatrix} < 0
\]
Theorem: There exists a controller such that the analysis LMI is satisfied if and only if there exists structured $Y$ and $X$ such that

$$
U^* \begin{bmatrix}
AY + YA^* & YC_1^* & B_1 \\
C_1Y & -I & D_{11} \\
B_1^* & D_{11}^* & -I \\
\end{bmatrix} U < 0
$$

$$
V^* \begin{bmatrix}
A^*X +XA & XB_1 & C_1^* \\
B_1^*X & -I & D_{11}^* \\
C_1 & D_{11} & -I \\
\end{bmatrix} V < 0
$$

$$
\begin{bmatrix}
X_0 & I \\
I & Y_0 \\
\end{bmatrix} > 0
$$
Controller implementation:

\[
\begin{align*}
\dot{x}_T &= A_{TT}x_T + A_{TS}x_S + B_T y \\

\Delta_S x_S &= A_{ST}x_T + A_{SS}x_S + B_S y \\
u &= C_T x_T + C_S x_S + Dy
\end{align*}
\]

EX: 2D

\[
x_S = (x_1, x_{-1}, x_2, x_{-2})
\]
Decentralized Control
Decentralized Control

Distributed Control
COMPARISON OF SPATIAL 2-NORM, ROLL ANGLE
STRONG NONLINEAR COUPLING

Nonlinear Spatially Interconnected Systems:

\[
\Delta x = f(x) + g(x)d
\]
\[
z = h(x)
\]

- Feedback linearization
- Backstepping
- etc.
Spatially and Time Varying Systems:
- non-homogeneous properties
- finite boundary conditions

\[ \Delta x = A(s)x + B(s)d \]
\[ z = C(s)x + D(s)d \]

TOOLS:
- LTI to LTV machinery (GEIR DULLERUD, UIUC)
- method of images, etc.
- LPV tools
Framework for Robust Control of IC systems

$$\Delta x(t, s) = F(x(t, s), d(t, s), s)$$

$$z(t, s) = H(x(t, s), d(t, s), s)$$

Delta contains temporal operators, spatial operators, AND uncertainty.

**Student:** Ramu Chandra (AE)

**Model Reduction** (CAROLYN BECK, UIUC)

**Cross-Directional Control** (GREG STEWART, HONEYWELL)
Phased Array Antennas for AFV Communication

Student: Sean Breheny (ECE)

- High data rate comms between AFVs and base station/satellite (video, etc.)
- Difficult to put a high gain antenna on an AFV (size constraint)
- Since it may be advantageous to use groups of AFVs anyway, why not investigate whether a formation of AFVs, each carrying a low gain antenna, could form a high gain phased array?
What is a Phased Array Antenna?

- Exploit EM wave interference among several antennas.
- For the simplest case (where array elements are not strongly coupled to each other), gain increases roughly linearly in N, the number of elements.
- Channel capacity increases linearly when the maximum bandwidth is used.
Example: Endfire Array

10 Element Endfire Array, Nominal Gain=8.1
Red - Original, Blue - Uncorrected, Black - Corrected
NOTE: Radial axis is linear