Consider the two unity feedback loops shown below with the uncertainty in the first system given by $\Delta$ stable, $\|\Delta\|_\infty \leq 1$ and the $P$ stable, minimum phase, and $P$ and $P^{-1}$ both proper.

1. Give the conditions for robust stability of the system subject to the uncertainty $\Delta$. Be prepared to sketch your results.

2. Consider the performance specification given by $\|H_{ud}\|_\infty < 1$ for all $\Delta$, where $H_{ud}$ is the transfer function from $d$ to $u$. Show that a necessary and sufficient condition for robust performance is that $\|CS\|_\infty < 1/2$.

3. Which of the following properties of $P$ is necessary in order for the robust performance condition to hold?

   (i) $P$ stable
   (ii) $P$ minimum phase
   (iii) $P$ biproper

   Explain your answer.

4. Suppose now that $P$ has a right half plane pole at $s = a$, so that it is no longer stable. Describe the performance limitations on $H_{ud}$. 