1. Consider the matrix
\[ A = \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 2 \end{pmatrix}. \]
(a) Find the generalized eigenspaces for each eigenvalue of \( A \).
(b) Compute the exponential \( e^{tA} \).
(c) Solve the differential equations
\[ \begin{align*}
\dot{x} &= y \\
\dot{y} &= -x \\
\dot{z} &= 2z
\end{align*} \]
with the initial conditions \( x(0) = 1, y(0) = 0, z(0) = 2 \).

2. Suppose that the real \( n \times n \) matrix \( A \) satisfies \( A - A^T = B \), where \( A^T \) denotes the transpose of \( A \) and where \( B \) commutes with \( A \); that is, \( AB = BA \). Is \( A \) diagonalizable?

3. Is the solution \((0, 0)\) of the system of equations
\[ \begin{align*}
x + 3y + x^4 - y^4 &= 0 \\
y + x^3 - y^3 &= 0
\end{align*} \]
isolated? Prove or give a counterexample.

4. (a) Show that the following defines a norm on the space of \( n \times n \) real matrices:
\[ \| A \|^2 = \text{trace}(A A^T) \]
(b) Let \( R \) be a (given fixed) real orthogonal matrix and \( S \) a (given fixed) real symmetric matrix. Show that there is a unique real symmetric matrix \( B \) such that
\[ \frac{1}{2} R^T B R - S = B. \]