CDS 270 (Fall 09) - Assignment 2 (Due Tuesday, Oct. 13)

1. Show that the sublevel sets of convex functions are convex.

2. (Feabible sets of LMIs) For symmetric matrices, $A_0, A_1, \ldots, A_m \in \mathbb{S}^n$, a linear matrix inequality (LMI) is a constraint on $\mathbb{R}^m$ of the form

$$A_0 + \sum_{i=1}^{m} x_i A_i \succeq 0,$$

where the notation "$Q \succeq 0$" stands for "Q is positive semidefinite."

Show that the set

$$\left\{ x \in \mathbb{R}^m : A_0 + \sum_{i=1}^{m} x_i A_i \succeq 0 \right\}$$

is convex.

3. Show that the following functions are convex:

   (a) $f(x, y) = \frac{x^2}{y}$ on the domain $\{(x, y) \in \mathbb{R}^2 : y > 0\}$

   (b) $f(x) = \frac{\|Ax+b\|_2^2}{c^T x+d}$ on the domain $\{x \in \mathbb{R}^n : c^T x + d > 0\}$