ESE 605-001: Modern Convex Optimization		**Spring 2021**
	Homework 3	
Assigned: 03/02/2021		*Due: 03/12/2021*

Homework must be LaTeX'd or it will not be graded.

You are expected to provide complete and rigorous solutions to all questions. Do not leave portions of your solutions as "exercises for the grader." If you use external resources please be sure to cite them.

CVX* warmup: Read the documentation for your choice of CVX* (see `https://canvas.upenn.edu/courses/1569651/pages/cvx-star-resources`) and implement **one of the following** based on your choice of coding language:

- CVX (Matlab): the optimization problem found on the landing page
- CVXPY (Python): the optimization problem found on the SOCP example page
- Convex.jl (Julia): the optimization problem found on the quick tutorial page

Please submit both your code as well as the output of the optimal value and solution to your problem.

Problems from Boyd & Vandenberghe: 4.28, 4.39, 4.43(a-c), 4.45

Disciplined Convex Programming: Read the slides posted to canvas that present an overview of how CVX* implements Disciplined Convex Programming (DCP). DCP is essentially a software implementation of the vector-valued composition rules that we have seen in class, and is what allows CVX* to recognize valid convex problem formulations. Then, apply your new found prowess to by solving Additional Exercise A3.3.

Problems from Boyd & Vandenberghe: 5.5, 5.7

Bonus questions: Bonus questions are completely optional. If a certain threshold of correctness is exceeded, you will earn up to an additional 2 marks per question on your assignment grade. These are fun, challenging problems, and we ask that you try your best to get as far into the proofs/answers as you can *without consulting any outside sources!* Once you get stuck, indicate the point at which you were stuck in your solutions with a "**I made it this far on my own**," after which, you should indicate what outside source you consulted in order to finish the problem, in accordance with the Penn Academic Integrity policy.

1. B&V 4.26(b)
2. B&V 4.43(d)
3. (S-Lemma, worth 4 points): Let $A, B \in \mathbb{S}^n$, and assume that there exists $\bar{x} \in \mathbb{R}^n$ such that $\bar{x}^\top A \bar{x} > 0$. Then $x^\top Ax \geq 0 \implies x^\top Bx \geq 0$ if and only if there exists a nonnegative λ such that $B \succeq \lambda A$. Note that the "if" direction here is very easy, and what needs to be proved is the "only if" direction. Hint: it suffices to demonstrate that if the optimization problem

$$\min_x \{x^\top Bx \,:\, x^\top Ax \geq 0\}$$

is strictly feasible with optimal value greater than or equal to 0, then $B \succeq \lambda A$ for some $\lambda \geq 0$.