Configuration Control of Non-Colliding Agents

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***Abstract*— A group of agents is required to achieve a *desired final configuration* or *shape* while avoiding collision. We design control laws that accomplish this task based on potential methods. The controller requires the agents to lie on a desired target manifold, which is assumed to be given by the zero set of a function with certain property. The collision avoidance scheme is local in nature, that is, it restricts the communication to neighboring agents which ensures numerical efficiency.**

I. INTRODUCTION

This paper considers the problem of controlling n agents, where n is large $\in \mathbb{N}$, to achieve a desired final *configuration* or *shape*, as shown in Figure 1. More specifically, we design control laws that move the agents, while avoiding collisions, from a random initial configuration in $\mathbb{R}^3$ to form a final configuration in $\mathbb{M}$, where $\mathbb{M} \subset \mathbb{R}^3$ is a *desired target subset*.

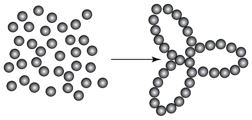

Fig. 1. Identical agents are controlled to achieve a desired shape or configuration. The control law moves the agents to the target region without requiring that a given agent be placed at a specified position.

A. Motivation and Related Work

Motion control of large systems of agents, also referred to as *agents*, is an active area of research with numerous applications in biology, chemistry, physics and engineering. For instance, understanding the swarming and schooling behavior in groups of fish, birds, bees, etc, has been the center of several recent studies; see, e.g., [7] and [14] for exposure to the animal biologists approach. Several mathematical models of the coordinated control of such systems have been proposed, see, e.g., [1], [2], [11], [12] and references therein for details. Engineering applications of the coordinated behavior of groups of agents include self-assembly of mobile agents, see [3]; formation control of groups of satellites, see, e.g., [4], [5], [9]; stabilization and coordination of Lagrangian systems, e.g., [8], [9]; massive distributed sensing which employs networks of mobile sensors in a given environment, such as the use of mobile underwater gliders for adaptive sampling of the ocean's thermal and/or chemical properties, [6].

This work addresses the problem of controlling a group of agents to achieve a *desired final configuration* or *shape* while avoiding collision among each other. That is, the focus of the control law is to reconfigure the agents to form a desired shape at a particular location in inertial space. This is in contrast to works focusing on the coordinated behavior of the agents as in [11], [16]. Another work which is close is spirit to this paper is [15] where the authors consider the problem of sampling an implicit surface by considering floater particles spread evenly across a surface using local repulsion.

B. The Control Problem

We consider n identical agents (of unit mass) in $\mathbb{R}^3$ and let $\boldsymbol{x}_i$ denote the position vector of a given agent i, $i = 1, \ldots, n$. The dynamics of each agent is assumed to be given by

$$\ddot{\boldsymbol{x}}_i = \boldsymbol{u}_i, \tag{1}$$

where the dot denotes derivative with respect to time t and $\boldsymbol{u}_i$ is the control force to be determined in order for the agents to form a desired shape or configuration, i.e., to lie on a desired subset $\mathbb{M} \subset \mathbb{R}^3$. One way to do this would be to define a distance function between the agent and $\mathbb{M}$ and dynamically flow along the negative gradient of this function. But this method can lead to ambiguities. For example, if the subset $\mathbb{M}$ is a sphere in $\mathbb{R}^3$ and an agent starts with zero initial velocity at the centre of the sphere, then there is no unique direction for the agent to pick at $t = 0$. If we ignore such "small" sets of ambiguous initial conditions, the method works but is computationally expensive as we need to solve a minimization problem at each time step of the numerical integration algorithm used for solving (1). To simplify computations, especially in the case of large number of agents, we assume that the target subset $\mathbb{M}$ can be represented as the zero set of a function $f(x, y, z)$, where (x, y, z) are cartesian coordinates of $\mathbb{R}^3$. Examples of such $\mathbb{M}$ are planes with $f = ax + by + cz - d$, ellipsoids with $f = \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} - 1$ and hyperboloids with $f = \frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} - 1$.

The control force $\boldsymbol{u}_i$ is to be chosen such that each agent i approaches the subset $\mathbb{M}$ as time goes to infinity. Additional control forces which emulate friction determines whether the agents "settle" down with zero velocity or continue to move on the subset $\mathbb{M}$, i.e., the agents have zero velocities in the direction normal to $\mathbb{M}$. We require that the agents do not collide with each other as they approach $\mathbb{M}$. We also require the dynamics of the controlled system to be *Hamiltonian*. The latter condition will enable us to use the Hamitonian function as a Lyapunov function to prove stability and collision avoidance.

C. Organization of the Paper

In Section II, we describe the control law, and propose three distinct methods for collision avoidance in Section III.

We provide collision avoidance proof of the control law with collision avoidance based on potential barriers in Section IV. The numerical implementation and a discussion of the results are presented in Section V.

II. PROBLEM FORMULATION

We consider the control law to be given by the sum of three components $\boldsymbol{u}_{iM}, \boldsymbol{u}_{iC}, \boldsymbol{u}_{iD}$ such that

$$\ddot{\boldsymbol{x}}_i = \boldsymbol{u}_{iM} + \boldsymbol{u}_{iC} + \boldsymbol{u}_{iD}, \tag{2}$$

where $\boldsymbol{u}_{iM}$ is responsible to make the agents approach the subset $\mathbb{M}$, $\boldsymbol{u}_{iC}$ makes sure the agents do not collide with each other at any instant of time, and $\boldsymbol{u}_{iD}$ is a damping term inserted to emulate friction. We consider two types of friction forces: one that acts in all directions; or one that only act in the direction tranverse to $\mathbb{M}$. In the former case, the velocity of each agent approaches zero asymptotically and in the latter case, only the velocity component in the direction transverse to $\mathbb{M}$ approaches zero.

Assumption 1: The set $\mathbb{M}$ can be represented as the zero set of a function $f(\boldsymbol{x})$ with the following property: there exists a positive real number $\alpha > 0$ such that

$$\text{if } f(\boldsymbol{x}) \leq \sqrt{\alpha} \text{ and } \nabla f(\boldsymbol{x}) = 0, \text{ then } \boldsymbol{x} \in \mathbb{M}.$$

We require the controlled dynamics to be Hamiltonian and to make each agent approach $\mathbb{M}$ as time goes to infinity, i.e., $\lim_{t\to\infty} f(x_i, y_i, z_i) = 0$, where (x_i, y_i, z_i) denote the components of the position vector $\boldsymbol{x}_i$ of agent i. We propose a suitable form of the control force in the absence of collision avoidance first, that is, for $\boldsymbol{u}_{iC} = 0$. In this case, we take $\boldsymbol{u}_{iM}$ to be derived from a potential function $V_i(x, y, z)$ which is minimum on $\mathbb{M}$, i.e., for the set of points (x, y, z) satisfying $f(x, y, z) = 0$. A natural choice for such a potential function is $V_i = \frac{1}{2}\left|f(x_i, y_i, z_i)\right|^2$. That is, take

$$\boldsymbol{u}_{iM} = -\frac{\partial V_i}{\partial \boldsymbol{x}_i} = -f\nabla f. \tag{3}$$

Now, introduce a damping term

$$\boldsymbol{u}_{iD} = -d_i\dot{\boldsymbol{x}}_i, \qquad \text{where } d_i \geq 0, \tag{4}$$

to ensure that the agents asymptotically approach $\mathbb{M}$ for all initial conditions starting from a bounded set. We make these statements more precise in the following theorem.

Theorem 2.1: Consider the motion of agent i described by (2) with $\boldsymbol{u}_{iC} = 0$ and $\boldsymbol{u}_{iM}, \boldsymbol{u}_{iD}$ given by (3) and (4) respectively. Let U denote the subset of the phase space given by:

$$U = \left\{ (\boldsymbol{x}, \dot{\boldsymbol{x}}) | E(\boldsymbol{x}, \dot{\boldsymbol{x}}) = \frac{1}{2}\dot{\boldsymbol{x}}^2 + \frac{1}{2}(f(\boldsymbol{x}))^2 \leq \frac{\alpha}{2} \right\}.$$

If the initial positions and velocities $(\boldsymbol{x}_i(t_0), \dot{\boldsymbol{x}}_i(t_0))$ of all agents are in U, then $(\boldsymbol{x}_i(t), \dot{\boldsymbol{x}}_i(t))$ remain in U for all time t, i.e., U is an invariant set and for these initial conditions. Further, all agents approach $\mathbb{M}$ asymptotically.

Proof. In the absence of collision avoidance, the dynamics of the agents are decoupled and we can focus on a single agent. The dynamics for the i^{th} agent is given by:

$$\ddot{\boldsymbol{x}}_i = -f\nabla f - d_i\dot{\boldsymbol{x}}_i. \tag{5}$$

Define the "energy" E_i of agent i to be the value of the energy-like function $E(\boldsymbol{x}, \dot{\boldsymbol{x}})$ evaluated at $(\boldsymbol{x}_i, \dot{\boldsymbol{x}}_i)$. The time derivative of $E_i = E(\boldsymbol{x}_i, \dot{\boldsymbol{x}}_i)$ is given by

$$\begin{aligned} \dot{E}_i &= \dot{\boldsymbol{x}}_i^T\ddot{\boldsymbol{x}}_i + f\dot{\boldsymbol{x}}_i^T\nabla f \\ &= \dot{\boldsymbol{x}}_i^T(-f\nabla f - d_i\dot{\boldsymbol{x}}_i) + f\dot{\boldsymbol{x}}_i^T\nabla f \\ &= -d_i\dot{\boldsymbol{x}}_i^T\dot{\boldsymbol{x}}_i \leq 0. \end{aligned} \tag{6}$$

That is, E_i is a nonincreasing function of time. If the initial conditions of agent i lies in U, then $(\boldsymbol{x}_i(t), \dot{\boldsymbol{x}}_i(t))$ remain in U for all time, i.e., U is an invariant set. Now, the Hamiltonian function for the system of agents corresponding to E_i is given by

$$H = \sum_{i=1}^{n}(E_i) = \sum_{i=1}^{n}\left(\frac{1}{2}\dot{\boldsymbol{x}}_i^2 + \frac{1}{2}(f(\boldsymbol{x}_i))^2\right). \tag{7}$$

One has $H \geq 0$ for all t by construction, but (6) implies that $\dot{H} \leq 0$. The only way this can be satisfied is if $\lim_{t\to\infty} H =$ constant and $\dot{H} = 0$. This implies that, as $t \longrightarrow \infty$, $\dot{\boldsymbol{x}}_i = 0$ for all i. Now, substitute $\dot{\boldsymbol{x}}_i = \ddot{\boldsymbol{x}}_i = 0$ in (5) to get

$$f\nabla f|_{\boldsymbol{x}_i} = 0. \tag{8}$$

If $f(\boldsymbol{x}_i) \neq 0$ and $\nabla f|_{\boldsymbol{x}_i} = 0$, we get a contradiction because $(\boldsymbol{x}_i, \dot{\boldsymbol{x}}_i) \in U$ means that $\boldsymbol{x}_i$ satisfies $f(\boldsymbol{x}_i) \leq \sqrt{\alpha}$. Hence, Assumption 1 is satisfied and $\boldsymbol{x}_i$ settles in $\mathbb{M}$. $\blacksquare$

Remark 2.2: For specific examples, it may be possible to choose f such that for almost all initial conditions, the individual agent asymptotically approach $\mathbb{M}$. Consider the case when $\mathbb{M}$ is a unit sphere given by the zero set of $f(\boldsymbol{x}) = \boldsymbol{x}^T\boldsymbol{x} - 1$. In this case, the only initial condition for which the agents do not approach $\mathbb{M}$ is when it starts with zero initial velocity at the origin.

III. COLLISION AVOIDANCE

In this section, we present three different ways to incorporate collision avoidance into our configuration control law. Namely, we choose $\boldsymbol{u}_{iC}$ to be (1) a repulsive centerline force, (2) a repulsive orthogonal force and (3) and a force derived from local potentials. Simulations suggest that the first two approaches achieve configuration control with collision avoidance for bounded agent velocities, but we do not present rigourous convergence proofs here. The third approach lends itself to an analytical proof when the agent velocities are bounded, which is a realistic assumption.

A. Repulsive Centerline Forces

Let $\boldsymbol{u}_{iC}$ be of the form:

$$\boldsymbol{u}_{iC} = \sum_{l=1}^{m} k_i(\boldsymbol{x}_i - \boldsymbol{x}_l) \tag{9}$$

Here, $k_i > 0$ and the summation is over all agents that are in a δ_i-neighbourhood of agent i. That is, when an agent l lies

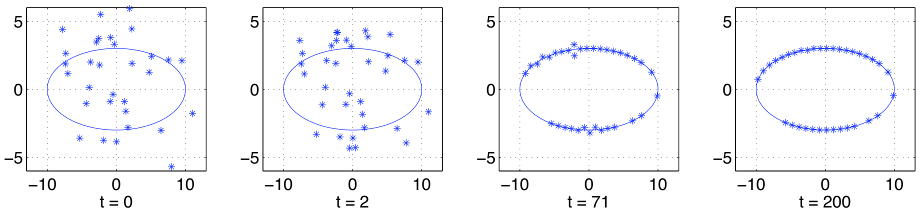

Fig. 2. Simulation of configuration control of 35 planar agents with the collision avoidance scheme based on repulsive forces presented in (9).

within a distance δ_i from agent i, both agents experience a repulsive force along the line joining them, with magnitude increasing as the distance increases. If there are m agents within the δ_i neighbourhood of agent i, agent i experiences a net force which amounts to the (weighted) sum of all repulsive forces.

The collision avoidance control law proposed in (9) seems counter-intuitive in the sense that the closer two agents are, the smaller the control force that repels them. It is more common to employ collision avoidance controls that tend to infinity as the inter-agent spacing tends to zero. Yet, the control law in (9) is worth presenting here since it appears to work well when the agents velocities are bounded. This can be explained as follows: as soon as two agents detect each other, $\boldsymbol{u}_{iC}$ works to separate them with a large force at the detection radius. For bounded velocities, this force is enough to make sure they do not collide even though the force decreases with smaller interspacing distance.

B. Repulsive Orthogonal Forces

We now consider the following repulsive force $\boldsymbol{u}_{iC}$:

$$\boldsymbol{u}_{iC} = \sum_{l=1}^{m} \hat{K}_i(\boldsymbol{x}_i - \boldsymbol{x}_l) \tag{10}$$

where $\hat{K}_i$ is a skew-symmetric constant matrix. This means that $\boldsymbol{u}_{iC}$ is perpendicular to $(\boldsymbol{x}_i - \boldsymbol{x}_j)$, hence the name orthogonal force. As before, the summation is over all agents that lie in a δ_i-neighbourhood of agent i. The above choice for $\boldsymbol{u}_{iC}$ means that when an agent l is within δ_i of agent i, they move away from each other along a line orthogonal to the line joining them. If there are more than one agent within a δ_i neighbourhood of agent i, then agent i experiences a repulsive force equal to the (weighted) sum of all repulsive forces.

Note that in the two dimensional setting, when we have two agents, the vector joining them and a vector orthogonal to it form a basis set. Hence, any collision avoidance scheme which tries to move away the agents has to be a combination of repulsive centerline and repulsive orthogonal forces.

The repulsive orthogonal forces are to be contrasted with gyroscopic forces used in [1]. It should be mentioned that gyroscopic forces are velocity dependent as opposed to repulsive orthogonal force which are only position dependent. A gyroscopic force that couples agent i and agent j takes the general form $\hat{G}\dot{\boldsymbol{x}}_i$, with $\hat{G}$ being a skew-symmetric (matrix-valued) function of $(\boldsymbol{x}_i, \boldsymbol{x}_j, \dot{\boldsymbol{x}}_i, \dot{\boldsymbol{x}}_j)$. Hence, gyroscopic forces are zero when velocities are zero. This means that if two agents are close to each other with zero velocities, gyroscopic forces will not move them away from each other as repulsive orthogonal forces does to keep a minimum interspacing distance. The collision avoidance controllers in this section can be thought of shaping the potential energy of the agents and its roots can be traced back to [13]. Whereas, the gyroscopic force based collision avoidance developed in [1] are based on shaping the part of the Lagrangian corresponding to the term linear in velocity. For yet another collision avoidance scheme based of shaping the term quadratic in velocities, see [10].

C. Repulsive Forces based on Potential Barriers

The idea here is to assign a potential barrier to each agent with a finite extinction radius. More specifically, we introduce a potential function φ_{ij} that denotes the potential barrier of the i^{th} agent seen by j^{th} agent:

$$\varphi_{ij}(\boldsymbol{x}_j; c_i, \delta_i) = c_i \exp(\frac{2}{\delta_i})\phi(r_{ij} + \delta_i)\phi(\delta_i - r_{ij}), \tag{11}$$

where $r_{ij} = (x_j - x_i)^2 + (y_j - y_i)^2 + (z_j - z_i)^2$ and c_i, δ_i are constant parameters. The function ϕ is given by:

$$\phi(r) = \begin{cases} 0 & \text{if} \quad r \leq 0 \\ \exp(\frac{-1}{r}) & \text{if} \quad r > 0 \end{cases} . \tag{12}$$

The potential function φ_{ij} takes its maximum value of c_i at

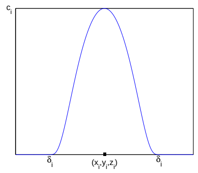

Fig. 4. The smooth function φ_{ij} attains its maximum c_i at (x_i, y_i, z_i) with a extinction radius of δ_i, i.e., $\varphi_{ij} = 0$ outside a δ_i neighbourhood.

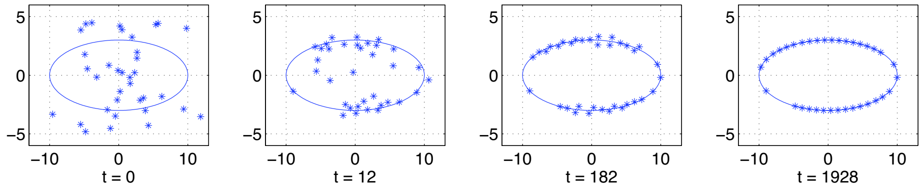

Fig. 3. Simulation of configuration control of 35 planar agents with the collision avoidance scheme based on repulsive orthogonal forces presented in (10).

(x_i, y_i, z_i) and is zero at distances greater than or equal to δ_i, as depicted in Figure 4. Further, it can readily shown that φ_{ij} is a smooth function and derivatives of all order vanish at distance $\geq \delta_i$. In other words, the potential barrier φ_{ij} of agent i affects all agents j, $j \neq i$, within a distance δ_i of (x_i, y_i, z_i). In some sense, the potential barrier collision avoidance force in this section can be seen as a smoothed version of the centerline repulsion collision avoidance force in Section III-A. Therefore, the control law for collision avoidance is local, i.e., each agents needs to know only the position of others within a radius δ_i.

The control force $\boldsymbol{u}_{iC}$ is:

$$\boldsymbol{u}_{iC} = -\sum_{j=1, j\neq i}^{n} \frac{\partial \varphi_{ji}}{\partial \boldsymbol{x}_i}, \tag{13}$$

where, in general, $\dfrac{\partial \varphi_{ij}}{\partial \boldsymbol{x}_j} \neq \dfrac{\partial \varphi_{ji}}{\partial \boldsymbol{x}_i}$. That is, the forces in $\boldsymbol{u}_{iC}$ and $\boldsymbol{u}_{jC}$ are not action reaction forces derived from a potential except in the special case where we assume $c_i = c_j$ and $\delta_i = \delta_j$ for all i, j. In that case, the controls $\boldsymbol{u}_{iC}$ are derived from a potential ϕ

$$\boldsymbol{u}_{iC} = -\frac{\partial \varphi}{\partial \boldsymbol{x}_i}, \qquad \text{where } \varphi = \sum_{1 \leq i < j \leq n}^{n} \varphi_{ij}. \tag{14}$$

IV. A STABILITY THEOREM FOR THE CONTROL LAW WITH COLLISION AVOIDANCE

In this section, we show that, for collision avoidance based on the potential barrier scheme, if the agents start in a properly chosen bounded set in the phase space, they stay within a bounded set without colliding with each other. Further, the velocity of each agent approaches zero asymptotically and we provide numerical evidence that the agents indeed approach the desired subset $\mathbb{M}$ asymptotically. We state the precise result in the following theorem.

Theorem 4.1: Consider the agents motion described by (2) with $\boldsymbol{u}_{iC}$ given by (14) and $\boldsymbol{u}_{iM}, \boldsymbol{u}_{iD}$ given by (3) and (4) respectively. Consider the subset of the phase space defined as:

$$\tilde{U} = \{(\boldsymbol{x}, \dot{\boldsymbol{x}}) | E(\boldsymbol{x}, \dot{\boldsymbol{x}}) \leq \frac{\alpha}{2n}\}.$$

Let the initial positions and velocities $(\boldsymbol{x}_i(t_0), \dot{\boldsymbol{x}}_i(t_0))$ for all agents be in $\tilde{U}$. Also, assume that, initially, the agents lie outside each other's individual tolerance distance δ_i, i.e., $\varphi_{ij}|_{t_0} = 0$. Then the individual trajectories $(\boldsymbol{x}_i(t), \dot{\boldsymbol{x}}_i(t))$ remain in the bounded set U defined above in Theorem 2.1 and $\dot{\boldsymbol{x}}_i$ approaches zero asymptotically. Further, if the parameters c_i in (11) are chosen such that $c_i > \alpha/2$ for each $i \in 1, \ldots, n$, then the agents do not collide with each other for all time t.

Proof. The equations of motion are

$$\ddot{\boldsymbol{x}}_i = -f(\boldsymbol{x}_i)\nabla f - d_i \dot{\boldsymbol{x}}_i - \frac{\partial \varphi}{\partial \boldsymbol{x}_i} \tag{15}$$

The Hamiltonian H_2 of the system in (15) is given by

$$H_2 = \sum_{i=1}^{n} \left(\frac{1}{2} (f(\boldsymbol{x}_i)^2 + \frac{1}{2} d_i \dot{\boldsymbol{x}}_i^2 + \varphi \right), \tag{16}$$

whose time derivative is $\dot{H}_2 = \sum_{i=1}^{n} -d_i \dot{\boldsymbol{x}}_i^T \dot{\boldsymbol{x}}_i \leq 0$. Clearly, H_2 is decreasing in time but the total Hamiltonian at t_0 is bounded by $\alpha/2$, hence, $H_2 \leq \alpha/2$ for all time. Therefore, $E(\boldsymbol{x}_i, \dot{\boldsymbol{x}}_i) \leq H_2 \leq \alpha/2$, which means that $(\boldsymbol{x}_i(t), \dot{\boldsymbol{x}}_i(t))$ lies in U for all time and $\boldsymbol{x}_i(t)$ satisfies Assumption 1. Also, using the same argument as in Theorem 2.1, we can show that, for each i, the agent velocity $\dot{\boldsymbol{x}}_i \to 0$ as $t \to \infty$. Now, to make sure the agents do not collide, i.e., φ_{ij} does not attain its maximum value $\alpha/2$, we only need to choose $c_i > \alpha/2$. Indeed, since H_2 is the sum of positive terms and is bounded above by $\alpha/2$, each φ_{ij} is bounded above by $\alpha/2$. Therefore, we have shown that the agents stay in U without colliding for all time and settle down to zero velocity asymptotically. ■

Remark 4.2: In the theorem above, the fact that $(\boldsymbol{x}_i(t), \dot{\boldsymbol{x}}_i(t))$ lies in U for all time and that $\dot{\boldsymbol{x}}_i \to 0$ asymptotically does not guarantee that the agents settle on $\mathbb{M}$. Indeed, one can have situations where the forces attracting the agents to $\mathbb{M}$ balance the collision avoidance forces as seen by the equilibrium condition:

$$0 = -f(x_i, y_i, z_i)\nabla f - \frac{\partial \varphi}{\partial \boldsymbol{x}_i}. \tag{17}$$

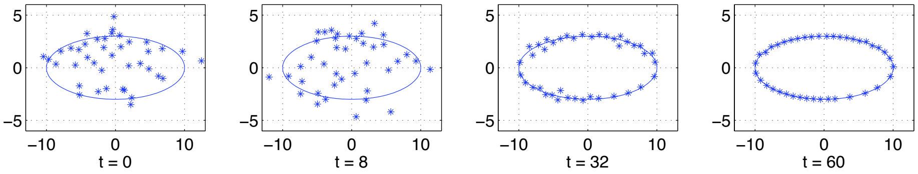

Fig. 5. Simulation of configuration control of 40 planar agents with the collision avoidance scheme based on potential barriers presented in (14).

However, these equilibria were never seen in simulations which suggests that they are all unstable. A rigorous proof of this claim remains to be undertaken.

Remark 4.3: Another interesting way in which the dissipation controls $\boldsymbol{u}_{iD}$ can be modified is as follows. Instead of choosing $\boldsymbol{u}_{iD} = -d_i\dot{\boldsymbol{x}}_i$, let $\boldsymbol{u}_{iD} = -\dot{f}(x_i, y_i, z_i)\nabla f$. This dissipation will not make the final velocities of the agents zero, but will only asymptotically drive them to the target manifold $\mathbb{M}$. Using this dissipation law, the agents asymptotically approach $\mathbb{M}$ and remain on $\mathbb{M}$ but with non-zero tangential velocities depending upon their initial conditions. That is, this dissipation asymptotically drives the agent velocities normal to $\mathbb{M}$ to zero.

V. Implementation and Numerical Results

We now present some simulation results for each of the collision avoidance schemes we presented in Section III. In all of the following cases, the initial positions were chosen randomly with a normal distribution and standard deviation of 5 units. The initial velocities were chosen to be zero for each agent. The goal in the simulation is to drive the agents without colliding with each other to an ellipse with center at the origin and major and minor axis lengths 10 and 3 units.

In Figure 2 is an illustration of a simulation that uses 35 planar agents and the collision avoidance control $\boldsymbol{u}_{iC}$ based on the repulsive centerline forces given in (9). The agents settle down after around 200 time units. Figure 3 illustrates a simulation of 35 planar agents with the collision avoidance control $\boldsymbol{u}_{iC}$ based on the repulsive orthogonal forces in (10). As can be seen, the agents take a much longer time to settle down in this case, around 2000 time units. Figure 5 illustrates a simulation with $\boldsymbol{u}_{iC}$ given by (14) for a system consisting of 40 planar agents. Even though we have larger number of agents here, the settling time turns out to be much lesser compared to the schemes presented in2 and 3. For the particular initial condition in the figure, the settling time was around 60 units.

The trend observed in Figures 2, 3 and 5 were consistent for all the random initial conditions we simulated. The settling time for the centerline repulsion method was at least twice as compared to potential barrier method and the settling time for orthogonal repulsion method was around an order of magnitude larger compared to potential barrier method. Further investigation needs to be carried to explain this behavior.

None of the initial conditions that we tried was attracted to the nontrivial equilibrium, i.e., $f(x_i, y_i, z_i) \neq 0$, given by (17), which suggests that these equilibria might be unstable. This can be verified heuristically for the example of two agents controlled to lie on an ellipse. In this example, we have three possible cases. In the first case, we have both the agents situated outside the ellipse at positions p_1 and p_2 as shown in Figure 7. This situation never arises if the tolerance distance for the individual agent δ_i is smaller than the smallest axis length of the ellipse. In the second case, we have one agent inside and one outside the ellipse. The only way this can happen is when both the agents lie in one of the four principal axis. See positions p_3 and p_4 in the figure. This scenario is possible even if δ_i is small but is unstable since a small perturbation of the positions of the agents moves them away from this equilibrium. In the third case, we have both the agents inside the ellipse. In this scenario, the force attracting an agent to the ellipse is in the same direction as the repelsion force which acts on this agent. Hence, there cannot be an equilibrium given by (17) in which both the agents are inside the ellipse. Therefore, for the case of two agents, the only nontrivial equilibrium given by (17) is unstable.

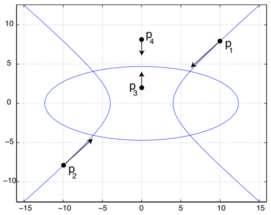

Fig. 7. Illustration of nontrivial unstable equilibria given by (17) for the case of two agents controlled to lie on an ellipse.

Before closing this section, we present an example with a more complex choice of subset $\mathbb{M}$. The agents are in a plane and the function f is chosen to be $f(x, y) = (x^2 + y^2)^2 - x^2 + y^2$. The zero level set of this function corresponds to a lemniscate in the plane. Figure 6 illustrates a system of 110 planar agents whose goal is to line up along the lemniscate

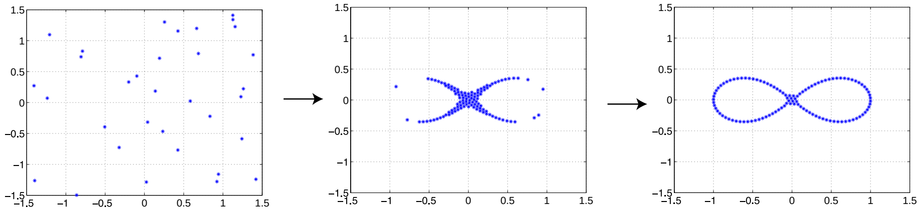

Fig. 6. Illustration of configuration control of 110 planar agents with collision avoidance. Note that the value of the detection radii $\delta_1, \ldots, \delta_n$, here chosen to be all equal, determine how many agents can be "accomodated" in the subset $\mathbb{M}$. For this simulation, the number of agents and their individual δ_i were chosen such that all of them could not be packed in the lemniscate. Hence, once sees clustering in the simulation around the origin.

given by $f = 0$ using the control law with the repulsive forces derived from potential barriers in (14).

VI. CONCLUSIONS

In this paper, we presented a method for the configuration control of a large number of agents with collision avoidance. The control law was designed to make the agents asymptotically approach a target set without specifying the final location of each agent. This is in contrast to the recent work in [8] where the communication topology is fixed throughout. The collision avoidance control was completely decentralized and each agent needed to communicate with only the agents that lied within its δ-neighborhood. We demonstrated in Section IV that under the assumption of finite bounded velocities of individual agents, we can choose controller parameters that guarantee stability of the control law with collision avoidance. We illustrated the output of this control law via a number of numerical experiments. We have numerical results for the case when the desired target manifold $\mathbb{M}$ is a slowly varying function of time, i.e., $\mathbb{M}_t \subset \mathbb{R}^3$ (these numerical experiments are not included in this papers), and the preliminary results seem to suggest that the controls derived in this paper are applicable. As part of our future work, we would like to extend the results in this paper to nonholonomic systems. In [10], potential based controllers are designed for underactuated hovercraft systems which guarantees that the system asymptotically goes to a neighbourhood of a target location. These controllers can be adapted to the setting in this paper to guarantee that these underactuated hovercraft lie in a neighbourhood of the desired target manifold. Finally, it is worth noting that these controls seem to be robust to potential errors done by the agents in estimating their relative distances. That is, if ϵ_i is a bound on the distance error estimated by the sensors in agent i, then we can redefine φ_{ij} to $\tilde{\varphi}_{ij}(x_j, y_j, z_j; c_i, \delta_i) = \varphi_{ij}(x_j, y_j, z_j; c_i, \delta_i + \epsilon_i)$, which makes the collision avoidance controls derived using $\tilde{\varphi}_{ij}$ robust to distance estimation errors.

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