Coordinated Control Scheme for Multi-Agents Systems

Zhipu Jin

Department of Control and Dynamical Systems
California Institute of Technology

May 17, 2006 CDS 270-2, lecture 8-2

Outline

1. Goals and Background
   - Goals and Readings
   - The Basic Problem That We Studied
2. Stability Analysis of Coordinated Control
   - Using State Space Approach
   - Using Transfer Functions Approach
3. Double-Graph Strategy
   - Double-Graph Model and Control Strategy
   - Stability and Performance Analysis
   - Simulations and Experiments on MVWT

Goals and References

Goals

- Formulate the coordinated control of multi-agent systems.
- Stability and performance analysis of formation control.
- Double-graph control scheme.

Reference

- "Information Flow and Cooperative Control of Vehicle Formations", J. Alexander Fax and Richard M. Murray, IEEE T. Automatic Control, 49(9):1465-1476, 2004;
- "Double-Graph Control Strategy of Multi-Vehicle Formations", Z. Jin and R. M. Murray, the 43rd IEEE Conference on Decision and Control, Dec. 14-17, 2004, Paradise Island, Bahamas.

Goals and References

Goals

- Formulate the coordinated control of multi-agent systems.
- Stability and performance analysis of formation control.
- Double-graph control scheme.

Reference

- "Information Flow and Cooperative Control of Vehicle Formations", J. Alexander Fax and Richard M. Murray, IEEE T. Automatic Control, 49(9):1465-1476, 2004;
- "Double-Graph Control Strategy of Multi-Vehicle Formations", Z. Jin and R. M. Murray, the 43rd IEEE Conference on Decision and Control, Dec. 14-17, 2004, Paradise Island, Bahamas.

Distributed Coordinated Control Strategy

Assumptions

- Agents are decoupled and have identical dynamics (LTI) in physical level.
- Local controllers have same structures and can exchange states by onboard sensors or communication channels.

Objectives

- Stability conditions with respect to local controller, interactive topology, and information flow.
- Performance evaluation (leader-follower case).

Distributed Coordinated Control Strategy

Assumptions

- Agents are decoupled and have identical dynamics (LTI) in physical level.
- Local controllers have same structures and can exchange states by onboard sensors or communication channels.

Objectives

- Stability conditions with respect to local controller, interactive topology, and information flow.
- Performance evaluation (leader-follower case).

Where did such problems come from?

- Understanding the group behavior of wild animals.


- Formation Control (Automated highway systems and UAVs).


Dynamics of Formation Motion

- A set of N agents whose (identical) linear dynamics are:

$$\dot{x}_i = Ax_i + Bu_i;$$

- Each agent's sensed information is defined as:

$$\begin{aligned} y_i &= C_1 x_i \\ z_{ij} &= C_2(x_i - x_j - d_{ij}), j \in N_i; \end{aligned}$$

- Decentralized controller:

$$\begin{aligned} \dot{v}_i &= K_A v_i + K_{B1} y_i + K_{B2} z_i \\ u_i &= K_C v_i + K_{D1} y_i + K_{D2} z_i \end{aligned}$$

where $z_i = \sum z_{ij}/|N_i|$.

Stability Theorem

Theorem

A local controller stabilizes the formation dynamics if and only if it simultaneously stabilizes the set of N systems:

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= C_1 x \\ z &= \lambda_i C_2 x \end{aligned}$$

where λ_i are the eigenvalues of $D^{-1}L$.

Proof

Using Kronecker matrix product and Schur decomposition.

Dynamics of Formation Motion

- A set of N agents whose (identical) linear SISO transfer functions are $P(s)$ and local controller are $C(s)$.
- For each agent, we have

$$\begin{aligned} H(s) &= \frac{P(s)C(s)}{1+P(s)C(s)} \\ y_i(s) &= H(s) \cdot \sum_j y_{ij}(s)/|N_i|; \end{aligned}$$

- The whole system is a MIMO system:

$$\begin{bmatrix} y_1(s) \\ y_2(s) \\ \vdots \\ y_n(s) \end{bmatrix} = (I - H(s) \cdot \Theta)^{-1} \cdot H(s) \cdot \begin{bmatrix} u_1(s) \\ u_2(s) \\ \vdots \\ u_N(s) \end{bmatrix}$$

where $\Theta = D^{-1}A$ is the average adjacency matrix.

Stability Theorem

Theorem

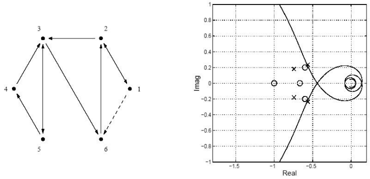
The distributed local controller stabilizes the formation dynamics iff the net encirclement of $-\lambda_i^{-1}$ by the Nyquist plot of P(s)C(s) is zero for all nonzero λ_i.

Proof

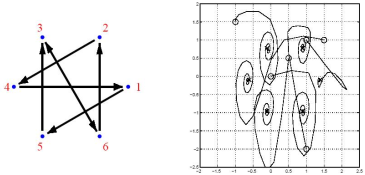
The determinant is: $\det(I - H(s)\Theta) = \prod_{i=1}^{N}(1 - \lambda_{\Theta i}H(s))$;
The char. poly. is: $\prod_i(1 + (1 - \lambda_{\Theta i})P(s)C(s))$;
Critical point -1 changes to multiple critical points $-(1 - \lambda_{\Theta i})^{-1}$.

Performance

- Limited stability region



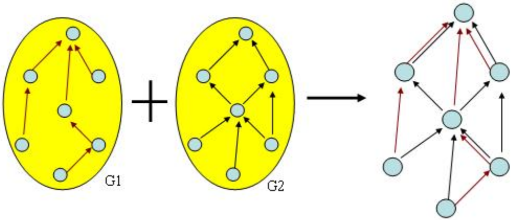
- Slow convergence speed



- Unscalable overshoots (string stability)

Double-Graph Model of a leader-follower system

- It's important for each follower that adjust their motions according to the leader as well as its neighbors.
- A connected digraph $\mathcal{G}_1$ is used to describe the information flow of group agreement (comm. network).
- Another connected digraph $\mathcal{G}_2$ is used to describe the neighbor states flow (onboard sensor).



Local Controller Structure

Transfer function of local controller

$$Y_i(s) = H_1(s) \cdot U(s) + H_2(s) \cdot \sum_j Y_{ij}(s)$$

where Y_{ij} is the outputs of vehicle i's neighbors in $\mathcal{G}_2$.

Simplify the analysis

$$\begin{aligned} H_1(s) &= \alpha \cdot H(s) \\ H_2(s) &= \frac{(1-\alpha)}{\Phi_i} \cdot H(s) \end{aligned}$$

where Φ_i is the out-degree of vehicle$i$ in $\mathcal{G}_2$ and $0 \leq \alpha \leq 1$.

Local Controller Structure

Transfer function of local controller

$$Y_i(s) = H_1(s) \cdot U(s) + H_2(s) \cdot \sum_j Y_{ij}(s)$$

where Y_{ij} is the outputs of vehicle i's neighbors in $\mathcal{G}_2$.

Simplify the analysis

$$\begin{aligned} H_1(s) &= \alpha \cdot H(s) \\ H_2(s) &= \frac{(1-\alpha)}{\Phi_i} \cdot H(s) \end{aligned}$$

where Φ_i is the out-degree of vehicle$i$ in $\mathcal{G}_2$ and $0 \le \alpha \le 1$.

Stability Analysis

Transfer function matrix

$$\begin{bmatrix} y_1(s) \\ y_2(s) \\ \vdots \\ y_n(s) \end{bmatrix} = (I - (1-\alpha) \cdot H(s) \cdot \Theta_2)^{-1} \cdot \begin{bmatrix} 1 \\ \alpha \\ \vdots \\ \alpha \end{bmatrix} \cdot H(s)U(s).$$

where $U(s)$ is the input of the leader.

Normalized adjacency matrix

A_2 is the adjacency matrix of the graph $\mathcal{G}_2$ and D_2 is a diagonal matrix with the out-degree of each node along the diagonal. Normalized adjacency matrix Θ_2 is defined by $\Theta_2 = D_2^{-1} \cdot A_2$.

Stability Analysis

Transfer function matrix

$$\begin{bmatrix} y_1(s) \\ y_2(s) \\ \vdots \\ y_n(s) \end{bmatrix} = (I - (1-\alpha) \cdot H(s) \cdot \Theta_2)^{-1} \cdot \begin{bmatrix} 1 \\ \alpha \\ \vdots \\ \alpha \end{bmatrix} \cdot H(s)U(s).$$

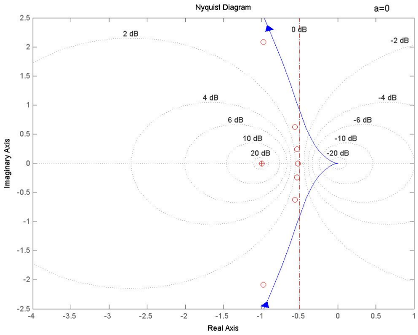
where $U(s)$ is the input of the leader.

Normalized adjacency matrix

A_2 is the adjacency matrix of the graph $\mathcal{G}_2$ and D_2 is a diagonal matrix with the out-degree of each node along the diagonal. Normalized adjacency matrix Θ_2 is defined by $\Theta_2 = D_2^{-1} \cdot A_2$.

Stability Analysis

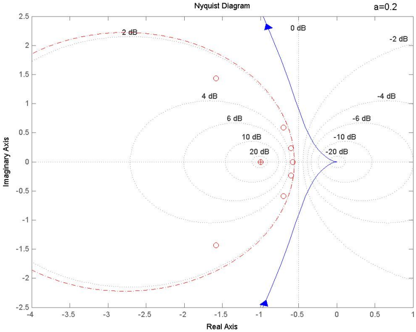
Nyquist plot, $\alpha = 0$



Critical point changed from -1 to several points. Determined by topology of $\mathcal{G}_2$ and α.

Stability Analysis

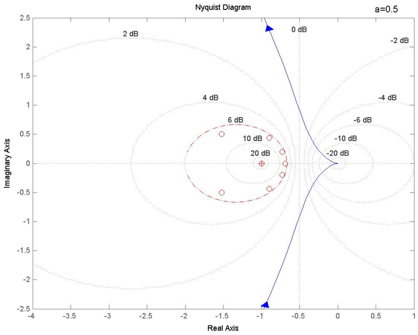
Nyquist plot, $\alpha = 0.2$



Critical point changed from -1 to several points. Determined by topology of $\mathcal{G}_2$ and α.

Stability Analysis

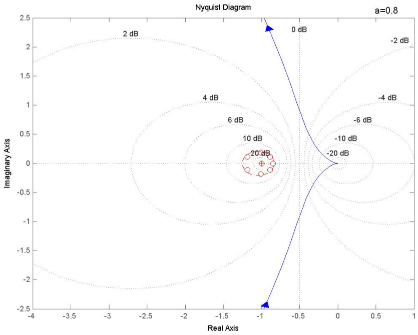
Nyquist plot, $\alpha = 0.5$



Critical point changed from -1 to several points. Determined by topology of $\mathcal{G}_2$ and α.

Stability Analysis

Nyquist plot, $\alpha = 0.8$



Critical point changed from -1 to several points. Determined by topology of $\mathcal{G}_2$ and α.

Performance Analysis

Theorem

For a leader-follower system with double-graph strategy, if $\|H(s)\|_\infty < 1/(1-\alpha)$*, then for any disturbance introduced at the leader, we have*

$$\|e_i(s)\|_2 < \|e_1(s)\|_2$$

where $e_i(s)$ *is the perturbation signal of the follower* $i \in [2, n]$*,* $e_1(s)$ *is the perturbation signal of the leader.*

Comments

The theorem solves the general “string stability” problem for any graph topology (or formation topology).

Performance Analysis

Theorem

For a leader-follower system with double-graph strategy, if $\|H(s)\|_\infty < 1/(1-\alpha)$, then for any disturbance introduced at the leader, we have

$$\|e_i(s)\|_2 < \|e_1(s)\|_2$$

where $e_i(s)$ is the perturbation signal of the follower $i \in [2, n]$, $e_1(s)$ is the perturbation signal of the leader.

Comments

The theorem solves the general “string stability” problem for any graph topology (or formation topology).

Performance Analysis

Definition

Treat the formation as an MIMO system and

$$\begin{cases} \|\underline{E}\|_\infty = \max_{i=1}^{n} \left(\|e_i(s)\|_2\right) \\ \|\underline{V}\|_\infty = \max_{i=1}^{n} \left(\|v_i(s)\|_2\right). \end{cases}$$

Theorem

For a leader-follower formation with double-graph strategy, If $\|(1-\alpha)\cdot H(s)\|_\infty = M < 1$, then

$$\frac{\|\underline{E}\|_\infty}{\|\underline{V}\|_\infty} \le \left(\frac{1-M^n}{1-M} + \frac{\rho(\Theta_2)M^n}{1-\rho(\Theta_2)M}\right)\cdot \|N(s)\|_\infty$$

where $\rho(\Theta_2)$ is the spectral radius of Θ_2 and $N(s) = \frac{e_1(s)}{v_1(s)}$.

Performance Analysis

Definition

Treat the formation as an MIMO system and

$$\begin{cases} \|\underline{E}\|_\infty = \max_{i=1}^{n} \left(\|e_i(s)\|_2\right) \\ \|\underline{V}\|_\infty = \max_{i=1}^{n} \left(\|v_i(s)\|_2\right). \end{cases}$$

Theorem

For a leader-follower formation with double-graph strategy, If $\|(1-\alpha)\cdot H(s)\|_\infty = M < 1$, *then*

$$\frac{\|\underline{E}\|_\infty}{\|\underline{V}\|_\infty} \leq \left(\frac{1-M^n}{1-M} + \frac{\rho(\Theta_2)M^n}{1-\rho(\Theta_2)M}\right)\cdot \|N(s)\|_\infty$$

where $\rho(\Theta_2)$ *is the spectral radius of* Θ_2 *and* $N(s) = \frac{e_1(s)}{v_1(s)}$.

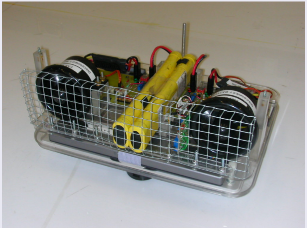
Trade Off in This Strategy

- So far, $\alpha = 1$ is the best choice for the stability issue and disturbance gains.
- Graph $\mathcal{G}_2$ provides the separation and cohesion feedbacks to keep the formation in certain shape. $\alpha = 0$ is good for this issue.
- There exists a trade off between the stability, disturbance resistance, and formation maintenance. The weight coefficient α is the indicator.

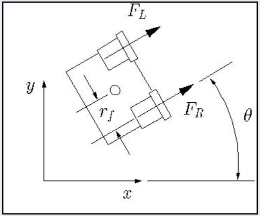
Quick review of Caltech MVWT

The Caltech Multi-Vehicle Wireless Testbed (MVWT) is an experimental platform for validating theoretical advances in multi-vehicle coordination and cooperation control.

Vehicle Kelly



Dynamics

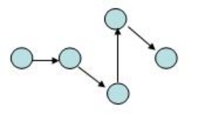


$$
\begin{aligned}
m\ddot{x} &= -\mu\dot{x} + (F_L + F_R)\cos\theta \\
m\ddot{y} &= -\mu\dot{y} + (F_L + F_R)\sin\theta \\
m\ddot{\theta} &= -\psi\dot{\theta} + (F_L - F_R)r_f
\end{aligned}
$$

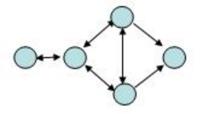
Simulations for different $\mathcal{G}_2$

Without double-graph strategy, or $\alpha = 0$

Topology 1



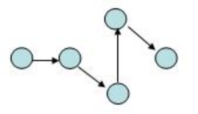
Topology 2



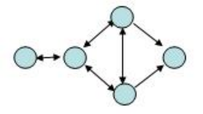
Simulations for different $\mathcal{G}_2$

With double-graph strategy, and $\alpha = 0.5$

Topology 1



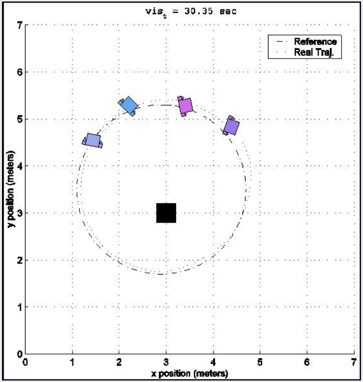
Topology 2



Four vehicle platoon experiment on MVWT

With double-graph strategy, and $\alpha = 0.6$

Experiment diagram



Experiment movie

Summary

- The stability of multi-agent systems is much complicated than consensus problem because of the agent dynamics. Local controller design is related to the interactive topology.
- The Double-graph strategy can greatly improve the stability and performance of large scale vehicle group. The cost we need to pay is an additional wireless communication network.
- Possible project ideas
  - Switch between different topologies and strategies (hybrid system).
  - Consensus problem in communication graph.