Problem Set #2

Reading: Abraham, Marsden, and Ratiu (MTA):

- Review Sections 2.1 and 2.2 (covered in CDS 201) as needed
- Read Sections 2.3–2.4
- Read Section 3.1–3.3

Problems:

1. MTA 2.3-1: Derivative of bilinear maps

2. MTA 2.3-4: Composition of a nonlinear and linear maps

3. MTA 3.1-4 (i) and (ii): Manifold structure of the Möbius band.
   
   Optional: Try to use your intuition about Möbius band to answer the following questions (then try them to see if you are right):
   
   - Consider a a Möbius band of finite width, like the one shown in Figure 3.4.4. What happens is you cut it down the center with a pair of scissors? Is the resulting set a manifold? Is it connected?
   - Repeat the experiment, but this time cutting the Möbius band one third of the distance from one of the edges.

4. MTA 3.1-5: Compactification of $\mathbb{R}^n$.

5. [Guillemin and Pollack, page 5, #3]
   
   Let $M$, $N$, and $P$ be smooth manifolds and let $f : M \to N$ and $g : N \to P$ be smooth maps.
   
   (a) Show that the composite map $g \circ f : M \to P$ is smooth.
   
   (b) Show that if $f$ and $g$ are diffeomorphisms, so is $g \circ f$.

   (You may use the fact that the composition of smooth functions between open subsets of Euclidian spaces are smooth.)

6. MTA 3.3-1: Graphs of manifolds