Nonlinear Control of Mechanical Systems in the Presence of Magnitude and Rate Saturations

Richard M. Murray

Mechanical Engineering
California Institute of Technology

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Abstract

This report summarizes the technical progress to date supported by this ONR Young Investigator Program (YIP) award. Accomplishments include new theoretical results on stabilization in the presence of feedback as well as preliminary results on trajectory generation in the presence of magnitude and rate limits. These results have been applied to an existing experimental apparatus and have been briefed to industry researchers.

1 Background

Linear control theory has become a highly developed and practical method of controlling dynamical systems. Over the past 30 years, techniques have emerged which allow the practicing engineer to quickly and efficiently generate linear controllers for a given system. These techniques include both analytic and software tools for designing controllers given a linear model of the plant and a structured description of plant uncertainties, external disturbances, and desired performance.

The fundamental assumption in linear control theory is that the dynamical system to be controlled can be approximately modeled as a linear system. This assumption is valid for a large class of systems, including many nonlinear systems operating in a neighborhood of an equilibrium point. Applications of linear control theory are widespread: feedback control of chemical processing systems, jet aircraft, and electro-mechanical devices have all benefited from the use of linear control tools.

For many systems, however, the fundamental assumption of a valid linear control model does not hold. High-performance jet aircraft exhibit strongly nonlinear dynamics due to the basic rigid body and aerodynamic forces active on the aircraft as well as nonlinearities in the systems which are used to actuate control surfaces and generate thrust. One of the most significant sources of nonlinearities is actuator saturation, which occurs in all modern control systems but is widely ignored by the existing analysis and synthesis tools.

Actuator saturation has a significant effect on the overall stability of aircraft. The recent YF-22 crash (April 1992) has been blamed on a pilot-induced oscillation (PIO) caused in part by time-delay effects introduced by rate saturation of control surfaces [6, 7]. As the complexity and performance of flight systems increase, stronger theoretical understanding is required to avoid such situations and guarantee performance of the system in the face of noise and unmodeled dynamics. These difficulties are already apparent in modern combat aircraft, which have multiple control surfaces that saturate at different magnitudes and rates and must be operated in a coordinated fashion in order achieve performance objectives over a large flight envelope. Without a strong basis for understanding the fundamental limitations and features of such systems, it will be difficult to exploit the full potential offered by active control of these systems.

Recent techniques in nonlinear control theory have begun to tackle some of the difficult issues involved in analysis of systems with input constraints. Nonlinear techniques by Teel, Sussmann, Sontag, and others have begun to show how nonlinear control laws can be used to achieve semi-global stability of linear and nonlinear systems in the presence of saturation. Recent advances in control of Lagrangian systems are providing new methods of utilizing the strong nonlinear structure which is present in mechanical systems and particularly motion control systems.

In this project we have begun to tackle the problem of magnitude and rate saturations by applying and extending some of these recent techniques and developing new methods tuned for flight control systems. There are many special properties of such systems which can be exploited in the search for practical techniques for nonlinear control in the presence of saturation. In additional to theoretical work on some of the fundamental issues involved in this problem, we are also carrying out experimental work on a small, vectored-thrust, flight control experiment at Caltech. This research will have direct application to vectored thrust aircraft, such as the F22, as well as more general air vehicles, such as ASTOVL aircraft and high performance missiles, which make use of advanced controls to achieve aggressive maneuvering and operation near the limits of the flight envelope.

The broad goals of this research are as follows:

1. To develop systematic techniques for analyzing and designing nonlinear control laws for mechanical systems with magnitude and rate constraints.
2. To develop synthesis tools which allow the use of local linear designs combined with global nonlinear methods to simultaneously achieve local robust performance and global robust stability.
3. To implement and test controllers on representative experiments which replicate important features of full-scale systems and to use these experiments to validate theoretical results and motivate new research directions.
4. To transition successful techniques into military and industrial applications by working with government and corporate partners to further develop methods initiated under this proposal, with the intent of possible implementation on full-scale aircraft.

2 Accomplishments

The accomplishments to date can be separated into two main areas: stabilization and trajectory generation. This separation is consistent with a two degree of freedom paradigm that has been proposed elsewhere [9].

2.1 Feedback stabilization

Consider a nonlinear control system

$$\begin{aligned} \dot{x} &= f(x,u) \\ y &= h(x) \end{aligned}$$

where the input u is restricted in both magnitude and rate:

$$|u| < L \qquad |\dot{u}| < M.$$

We wish to find a control law $u = \alpha(x, r(\cdot))$ such that the closed loop system is stable and the system tracks a reference input $r(t)$.

The first approach that we have developed makes use of a nonlinear gain scheduling technique that degrades the performance of the system when actuators are saturated in such a way as to insure stability. The form of the control law is

$$u = \alpha_\gamma(x, r(\cdot))$$

where γ is a scaling parameter that is adjusted based on whether the input saturates in magnitude or rate. Analytical results have been derived giving proof of convergence of the algorithm and indicating how and why the controller works [3].

Experimental results on the Caltech ducted fan show this approach to be very promising and these results will be presented at the AIAA Guidance, Navigation, and Control conference in August, 1997 [2].

More recent work has focused on strengthening our initial theoretical results to provide a better foundation for future progress [4]. Motivated by some recent results on the stabilization of homogeneous systems [8], we present a gain-scheduling approach for the stabilization of non-linear systems. Given a one-parameter family of stabilizing feedbacks and associated Lyapunov functions, we show how the parameter can be rescaled as a function of the state to give a new stabilizing controller. In the case of homogeneous systems, we obtain generalizations of some existing results. We show that this approach can also be applied to non-homogeneous systems. In particular, the main application considered in this paper is to the problem of stabilization with magnitude limitations. For this problem, we develop a design method for single-input controllable systems with eigenvalues in the left closed plane.

In addition, we have collaborated with Professor Andrew Teel at the University of Minnesota to test some of his controllers on our flight control experiment. Teel has developed techniques for combining local, high performance controllers with global, stabilizing controllers. These stabilizing controllers have poor performance, but can stabilize the system in the presence of actuator limits. As a consequence, it should be possible to achieve very good local performance using aggressive control designs while maintaining stability of the system at the operating envelope. Experimental testing of these controllers has demonstrated the efficacy of the approach as was recently presented at the 1997 American Control Conference [5].

2.2 Trajectory generation

While understanding the role of feedback in the presence of magnitude and rate limits is a challenging technical problem, a more relevant problem from the perspective of flight control is the trajectory generation problem. Motivated by applications in uninhabited combat aerial vehicles (UCAV), we have begun to build a foundation for real-time, trajectory generation in the presence of actuator constraints. These types of techniques will be essential for onboard generation of feasible trajectories that satisfy both the dynamics and the actuator constraints.

The approach that we have taken is based on the notion of differential flatness [11]. Trajectory generation for a system which is differentially flat can be reduced to an algebraic problem using basis functions in an appropriate space. In so doing, the dynamics of the system are reduced to a trivial system and the nominal inputs and states that satisfy the (unconstrained) dynamics can be determined in a computationally tractable way (see [11, 10] for details).

Our initial results in this area have focused on linear systems, which are a special case of differentially flat systems [1]. Given a linear system

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx, \end{aligned}$$

we give an algorithm for finding a feasible trajectory between a initial state x_0 and a final state x_f that satisifies the equations of motion and constraints on the input magnitude. The approach relies on approximating the constraints by a set of bounding hyperplanes and then finding a feasible point inside the volume bounded by these hyperplanes. This feasible point corresponds to a set of coefficients for the basis functions used to parameterize the trajectory space.

We plan to extend these results to the case of differentially flat systems.

3 Honors and Awards

10/96	Invited Speaker, United Technologies Advanced Studies Program
5/97	Plenary talk, SIAM Conference on Dynamical Systems and Their Applications
6/97	Donald P. Eckman Award (American Automatic Control Council)
10/97	Invited Speaker, United Technologies Advanced Studies Program
3/98	Member, Defense Science Study Group
5/98	Plenary talk, SIAM Conference on Control Theory and Its Applications

4 Personnel Supported

Name	Position	Comments
Richard M. Murray	PI	Partial salary support
Sunil Agrawal	Visiting Faculty	Partial salary support
Pascal Morin	Postdoc	Partial salary support
Herbert Streumper	Postdoc	Salary support
Scott Kelly	Graduate student	Salary support
Trygve Lauvdal	Visiting Student	Partial salary support
Oliver Kaiser	Visiting Student	Partial salary support

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