Two Degree of Freedom Design for Robust Nonlinear Control of Mechanical Systems

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Project Overview

- Computational methods for real-time trajectory generation for nonlinear mechanical sys-tems
- Theoretical and computational methods for analyzing robustness to perturbations which preserve the mechanical nature of the system
- Theoretical and experimental studies to ex-plore the tradeoffs between aggresiveness in trajectory generation and robust performance
- Implementation of proposed techniques in industrial applications

Configuration Controllability for Mechanical Systems

Motivating Problems

- For mechanical systems, often interested in moving between equilibrium points (or relative equilibria), not general states
- Requires more geometric understanding of controllability, tuned to mechanical systems

Objectives

- Give computable tests for controllability between equilibrium points rather than arbitrary states (configuration + velocity)
- Understand underlying geometry to exploit controllability structure in stabilization, trajectory generation, trajectory tracking

Techniques

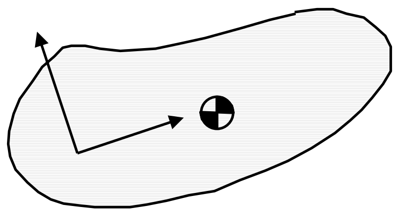
- Use Riemannian structure associated with kinetic energy metric (Levi-Cevita connection)
- Make use of *symmetric product* to encode geometry of geodesically invariant sets (complements Lie bracket)

Results to date

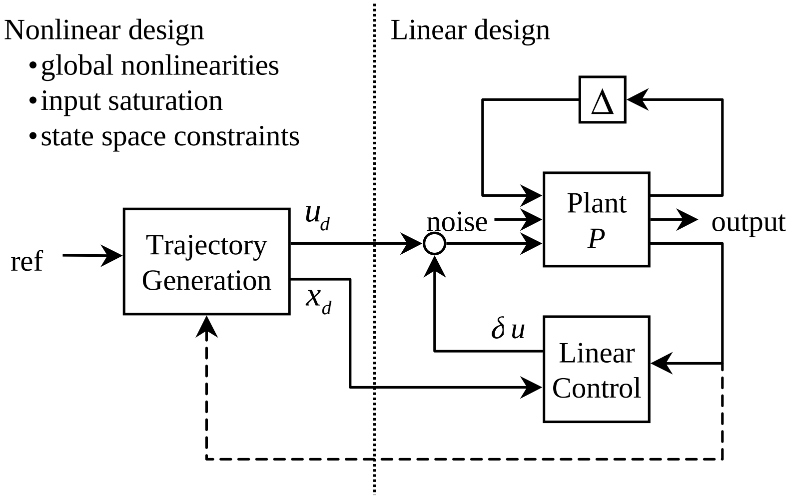
- Complete characterization of configuration accessibility (weak notion of controllability)
- Sufficient conditions for *equilibrium controllability* in terms of smallest "totally geodesic" distribution generated by external force fields

Significance

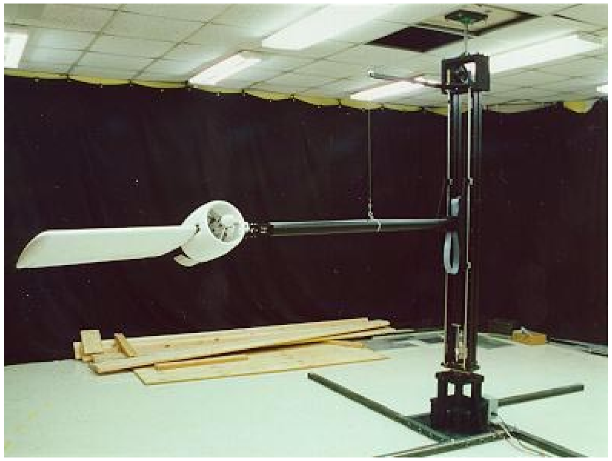
- Describes underlying geometry of nonlinear control of mechanical systems, exploit inherent structure (Lagrangian)
- Extensions to mechanical systems on Lie groups (e.g., satellites, aircraft, underwater vehicles) gives even sharper results (more structure)



Trajectory Generation and Tracking Using Differential Flatness



Caltech Ducted Fan



Approach: Two Degree of Freedom Design

- Use online trajectory generation to construct feasible trajectories
- Use (scheduled) linear control for local performance
- For many mechanical systems, system is *differentially flat* => reduce dynamic system to algebraic equivalent and generate feasible trajectories in real time

Results

- Aggressive tracking for fully actuated mechanical systems by exploiting mechanical structure (Bullo & Murray, 1997)
- Real-time algorithms developed and tested on ducted fan (Van Nieuwstadt and Murray, 1996)
- Necessary and sufficient conditions for flatness for a class of mechanical systems (Rathinam and Murray, 1996)

Summary of Accomplishments

Major Milestones

- General framework for control of mechanical systems, exploiting geometric structure [5]
- Necessary and sufficient conditions for differential flatness of mechanical systems with one fewer input than control [4]
- Application of flatness based control laws for tracking of aggressive trajectories to Caltech ducted fan [3]
- Algorithms for constructive controllability for underactuated mechanical systems on Lie groups [1]

Honors and Awards (1997)

- Plenary speaker, SIAM Conference on Applications of Dynamical Systems (May 97)
- Donald P. Eckman Award (AACC, June 97)
- Plenary speaker, SIAM Conference on Control and Its Applications (May 98)

Recent Papers

1. F. Bullo and N. Leonard. Motion Control for Underactuated Mechanical Systems on Lie Groups. 1997 European Control Conference.
2. F. Bullo and R. M. Murray. Trajectory tracking for fully actuated mechanical systems. To appear, *Automatica.*
3. M. van Nieuwstadt and R. M. Murray. Real time trajectory generation for differentially flat systems. To appear, *Int'l J. Robust and Nonlinear Control.*
4. M. Rathinam and R. M. Murray. Configuration flatness of Lagrangian systems underactuated by one control. To appear, *SIAM J. Control and Optimization.*
5. R. Murray. Nonlinear Control of Mechanical Systems: A Lagrangian Perspective. To appear, *Annual Reviews in Control.*