TWO DEGREE OF FREEDOM DESIGN FOR ROBUST NONLINEAR CONTROL OF MECHANICAL SYSTEMS

RICHARD M. MURRAY
CALIFORNIA INSISTUTE OF TECHNOLOGY

NFS CAREER Proposal, 1995

Motived by applications in flight control and robotics, this project focuses on the use of two degree of freedom design techniques to generate nonlinear controllers for mechanical systems performing motion control tasks. Sample applications include high-performance control of piloted aircraft using vectored thrust propulsion, navigation and control of unmanned flight vehicles performing surveillance and other tasks, motion control and stabilization of underwater vehicles and ships, and control of land-based robotic locomotion systems. This project makes use of experimental facilities currently available at Caltech as well as interaction with industrial projects to insure that research results are relevant to existing and future applications.

The basic approach of two degree of freedom design is to initially separate the nonlinear controller synthesis problem into design of a feasible trajectory for the nominal model of the system, followed by regulation around that trajectory using controllers that have guaranteed performance in the presence of uncertainties. This splitting of the problem offers several advantages over existing nonlinear methods and allows the use of advances in linear controller synthesis to help achieve robust performance.

In the applications which are considered, it is important that the trajectory generation phase be done in real-time and hence traditional techniques such as optimal control cannot be applied directly. Recent techniques in nonlinear control theory allow certain high-dimensional dynamic problems to be reduced to algebraic problems that are amenable to computationally efficient algorithms for trajectory generation. Using these ideas, new methods are being developed for quickly generating (suboptimal) trajectories which accomplish a desired control objective.

Another feature of the class of systems under consideration is their mechanical nature. Driven by new theoretical results in Lagrangian control systems, a more detailed understanding of the role of symmetries, constraints, and external forces is emerging which has important implications for the specific systems considered in this project. By properly accounting for the second order nature of mechanical systems, it is possible to analyze the control of these systems in a way which remains true to the underlying geometry and allows the nonlinear nature of the mechanical system to be exploited to a fuller extent than previously possible. This structure is being used both for trajectory generation and to understand the notion of robustness in the presence of physically meaningful model uncertainty and disturbances, such as uncertainty in dynamic and kinematic parameters and disturbance forces due to wind, fuel slosh, and actuator noise.

A major emphasis in this project is implementation of new techniques on existing experimental systems and development of software which can be used in other applications.

Research plan

1. Introduction

A large class of industrial and military control problems consist of planning and following a trajectory in the presence of noise and uncertainty. Examples range from unmanned and remotely piloted airplanes and submarines performing surveillance and inspection tasks, to mobile robots moving on factory floors, to multi-fingered robot hands performing inspection and manipulation tasks inside the human body under the control of a surgeon. All of these systems are highly nonlinear and demand accurate performance.

Modern geometric approaches to nonlinear control often rely on the use of feedback transformations to convert a system into a simplified form which can then be controlled with relatively standard techniques (such as linear feedback). While this approach very effectively exploits the nonlinear nature of the system, it often does so by "converting" the nonlinear system into a linear one. This can be disadvantageous if one is concerned with disturbance rejection and other performance specifications since the nonlinear transformations typically do not preserve many important properties of the system.

One way around this limitation is to make use of the notion of *two degree of freedom* controller design for nonlinear plants. Two degree of freedom controller design is a standard technique in linear control theory that separates a controller into a feedforward compensator and a feedback compensator. The feedforward compensator generates the nominal input required to track a given reference trajectory. The feedback compensator corrects for errors between the desired and actual trajectories. This is shown schematically in Figure 1.

Many modern nonlinear control methodologies can be viewed as synthesizing controllers which fall into this general framework. For example, traditional nonlinear trajectory tracking approaches, such as feedback linearization [12, 14] and nonlinear output regulation [13], are easily viewed as a feedforward piece and a feedback piece. Indeed, when the tracking error is small, the primary difference between the methods is the form of the error correction term: output regulation uses the linearization of the system about a single equilibrium point; feedback linearization uses a linear control law in an appropriate set of coordinates.

This two step approach can be carried one step further by completely decoupling the trajectory generation and asymptotic tracking problems. Given a desired output trajectory, we first construct a state space trajectory x_d and a nominal input u_d that satisfy the equations of motion. The error system can then be written as a time-varying control system in terms of the error, $e = x - x_d$. Under the assumption that that tracking error remains small, we can linearize this time-varying system about $e = 0$ and stabilize the $e = 0$ state. One method of doing this is to solve the linear quadratic optimal control problem to obtain the optimal (time-varying) feedback gain for the path. More advanced techniques include the use of linear-time varying robust synthesis (see, for example, Shamma [35] for recent results and a survey of the literature) and the use of linear parameter varying synthesis developed by Packard [32] and others.

The advantage of separating the trajectory generation and stabilization phases is that one can effectively exploit the geometric nature of the system to generate trajectories, while also making use of the linear structure of the error dynamics. Thus one can treat strong nonlinearities such as input saturation and global nonlinear behavior separately from the problem of robust stabilization and disturbance rejection along a reference trajectory.

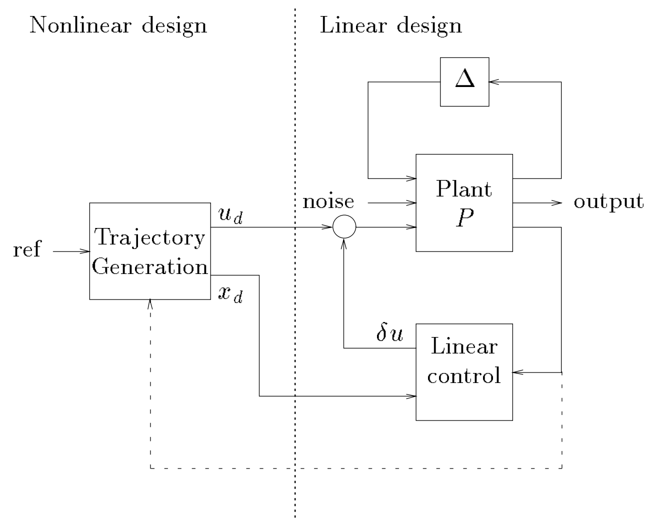

FIGURE 1. Two degree of freedom controller design.

2. Proposed Research

The broad goal of this project is to develop new engineering techniques for design of nonlinear controllers for mechanical systems performing motion control tasks. Existing and emerging applications in flight control, underwater autonomous vehicles, and land-based locomotion systems require performance which cannot be met using existing linear and nonlinear synthesis techniques. A common ingredient of many of these advanced applications is complicated dynamic behavior which must be *exploited* rather than ignored or treated as uncertainty. By making use of new advances in mathematics, computation, and control theory, we plan to address issues in robust nonlinear control of mechanical systems which arise in the context of several specific examples and generate software tools for building and analyzing nonlinear controllers for a broad class of mechanical systems.

We plan to initially concentrate on generation of trajectories for nonlinear systems, coupled with application of well-developed linear techniques for trajectory tracking. Our primary interest in this regard is in building real-time algorithms for trajectory generation that can be used in an experimental setting and that can be interfaced to existing linear synthesis techniques. This will be combined with more theoretical work in Lagrangian mechanics by designing algorithms which exploit the second order nature of mechanical systems and make use of symmetries and energy considerations which are specific to mechanical systems.

2.1. **Differentially flat systems.** One of the classes of systems for which trajectory generation is particularly easy are so-called differentially flat systems. Roughly speaking, a system is differentially flat if we can find a set of outputs (equal in number to the number of inputs) such that all states and inputs can be determined from these outputs without integration. More precisely, if the system has states $x \in \mathbb{R}^n$, and inputs $u \in \mathbb{R}^m$ then the system is flat if we can find outputs $y \in \mathbb{R}^m$ of the form

$$y = y(x, u, \dot{u}, \ldots, u^{(p)}) \quad \text{such that} \quad \begin{aligned} x &= x(y, \dot{y}, \ldots, y^{(q)}) \\ u &= u(y, \dot{y}, \ldots, y^{(q)}). \end{aligned} \tag{1}$$

Differentially flat systems are useful in situations where explicit trajectory generation is required. Since the behavior of flat system is determined by the flat outputs, we can plan trajectories in output space, and then map these to appropriate inputs.

Differentially flat systems were originally studied by Fliess et al. in the context of differential algebra [9] and later using Lie-Backlünd transformations [10]. In [41] we reinterpreted flatness in a differential geometric setting. We made extensive use of the tools offered by exterior differential systems and the ideas of Cartan. Using this framework we were able to recover most of the results currently available using the differential algebraic formulation and achieve a deeper geometric understanding of flatness. We also showed that differential flatness is more general than feedback linearization in the multi-input case. More importantly, the point of view is quite different, focusing on trajectories rather than feedback transformations.

2.2. **Mechanical systems.** The likelihood of building a general nonlinear theory which can be used for all nonlinear dynamical control systems is extremely small due to the diverse nature of behaviors that can be present in nonlinear systems. In this work, we plan to concentrate on one particularly well-structured class of nonlinear control systems: second order systems whose unforced motion is described by Lagrangian mechanics.

The are several reasons why this class of systems is a good candidate for new results in nonlinear control. On the practical end, mechanical systems are often quite well identified and very accurate models exist for specific systems, such as the High Altitude Research Vehicle (HARV) [4, 7]. Furthermore, instrumentation of mechanical systems is relatively easy to achieve and hence modern nonlinear techniques (which often rely on full state feedback) can be readily applied. On the theoretical end, new results in Lagrangian mechanics, motivated in large part by problems in control theory, have generated new insights into the control problem and have produced new techniques which have not been widely studied. These techniques exploit the symmetries, constraints, and energy properties of Lagrangian systems to understand the underlying behavior of the system.

Traditionally, work in geometric mechanics has ignored the role of constraints and external forces in the dynamics of the system (see, for example, Abraham and Marsden [1] or Arnol'd [2]). As a consequence, most current nonlinear control techniques do not make use of the extra structure available in mechanical systems. Recently, however, there has been substantial progress in geometric mechanics, motivated in large part by applications in nonlinear control theory (see Marsden [19] and Bloch et al. [3] for a surveys and examples). Much of the new theory has been in terms of Lagrangian mechanics (rather than Hamiltonian mechanics) and it appears that certain phenomena, such as constraint forces and external forces, are better dealt with in the Lagrangian framework [3].

One example of the application of this new theory is in the context of robotic locomotion [16]. For a large class of land-based locomotion systems—included legged robots, snake-like robots, and wheeled mobile robots—it is possible to model the motion of the system using the geometric phase associated with a connection on a principal bundle. By modeling the locomotion process using connections, it is possible to more fully understand the behavior of the system and in a variety of instances the analysis of the system is considerably simplified. In particular, this point of view seems to be well suited for studying issues of controllability and choice of gait.

Application of techniques from geometric mechanics to more general systems remains an open problem. However, there are many examples of systems where features common to geometric mechanics, such as translational and rotational symmetries, are present and there are strong indications that these features can be exploited in control design. The relationship between flatness and geometric mechanics is also intriguing and initial results in this direction are very encouraging. For example, for rigid bodies in the plane with body fixed forces (e.g. thrusters), it is possible to show that the system is flat using the xy position of the center of oscillation of the system. This hints at a deeper understanding of flatness for mechanical systems than for other systems where geometric quantities such as the center of oscillation are not defined.

Another area of interest is the use of Lagrangian structure to investigate robustness issues for control of mechanical systems. For linear control systems, there is a very rich theory of the robustness of control laws to structured perturbations (see, for example, [33]). For nonlinear systems no such general theory exists and it is not clear what type of structure can be placed on the perturbations. However, for mechanical systems much more is known. For example, perturbations must still preserve the overall second order nature of a mechanical system and even unmodeled dynamics can only enter the equations of motion in certain ways. Very little work has been done on this problem and we plan to explore these issues as part of this project.

3. Applications

The work described in this proposal has application to a wide variety of engineering systems. In this section we describe several such applications, concentrating on two with which the PI is currently involved.

3.1. **Unmanned Autonomous Flight Vehicles.** The use of unmanned flight vehicles for military and commercial surveillance tasks is increasing as demand for information and the costs associated with piloted surveillance techniques grow. As an indication of this trend, Northrop Grumman Corporation has recently received funding for a novel sensing system known as ROSS (Remotely Operated Sensor System). The ROSS program involves an aircraft flying in a circular pattern while towing a cable with a tow body (drogue) housing the sensor system attached at the bottom. Under suitable conditions, the cable reaches a relative equilibrium in which the cable maintains its shape as it rotates. By choosing the parameters of the system appropriately, it is possible to make the radius at the bottom of the cable much smaller than the radius at the top of the cable. This is illustrated in Figure 2. Initial flight tests have been run showing the feasibility of this method for remote sensing and work is currently underway to build a prototype system for use in surveillance tasks.

The dynamics of this system were studied extensively in the 1970s in research projects funded in large part by the Navy [36, 34]. Viewed as a mechanical system, the towed cable problem has a number of special properties that can be exploited. In the absence of wind, there is a natural symmetry of the system corresponding to rotation around the vertical axis. Furthermore, there are multiple equilibria for the system and bifurcations of these

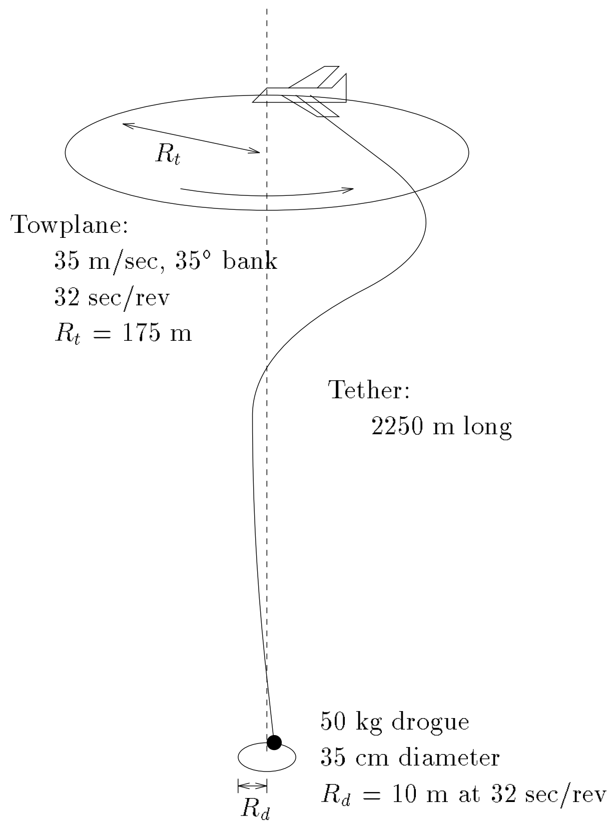

FIGURE 2. Relative equilibrium of a cable towed in a circle.

equilibria can occur when the cable length changes (which happens when the drogue is initially deployed). Wind acts as a symmetry breaking disturbance and modern techniques in structural stability are important for understanding the behavior of the system in such instances.

An important problem is generating trajectories for this system which allow transition between one equilibrium point and another with minimal settling time. Due to the high dimension of the model for the system (128 states is typical), traditional approaches to solving this problem, such as optimal control theory, cannot be easily applied. However, it can be shown that this system is differentially flat, using the position of the bottom of the cable as the differentially flat output. Thus all feasible trajectories for the system are characterized by the trajectory of the bottom of the cable.

In principle, this allows us to reduce the problem of trajectory generation for the system to moving the bottom of the cable along a path from the initial to final configurations. In doing so, we can reduce the original optimal control problem to an optimization problem (by choosing a finite parameterization of the flat outputs). However, there are still a number of constraints on the system which complicate the problem. The chief constraint is the limited performance of the towplane. It has maximum bank angles, minimum air speeds, and limits on acceleration and deceleration which must be observed. In the flat output space, these

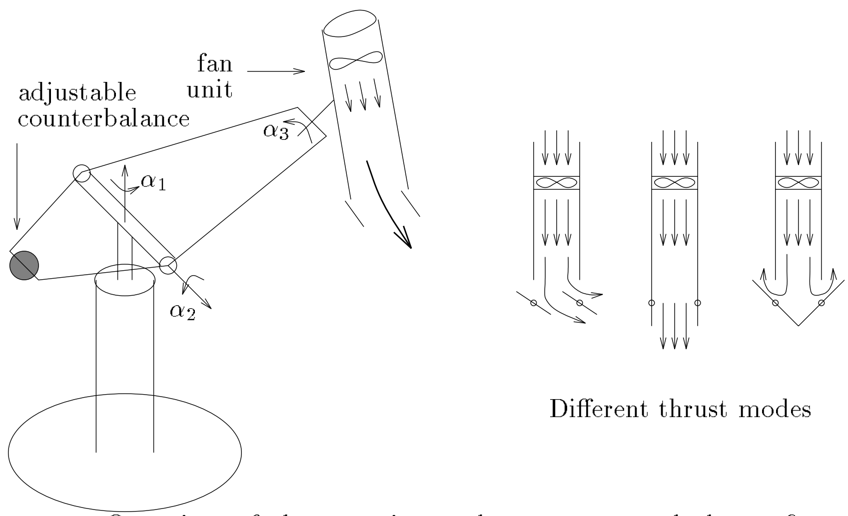

FIGURE 3. Overview of the experimental apparatus and thrust figures for different flap settings

constraints become constraints on higher order derivatives of the output and must be taken into account. However, the problem remains algebraically and computationally tractable.

Other issues of concern are the effects of noise (for example wind and turbulence) on the performance of the system and the role of unmodeled dynamics and parameter uncertainty. All of these issues are very difficult to handle using traditional techniques since the behavior of the system is highly nonlinear and the dynamics are high dimensional. The two degree of freedom technique proposed above, combined with the use of differential flatness for generating trajectories and mechanical properties for studying robustness, is exactly the type of technology which is needed to study this system and other unmanned flight control systems with complicated nonlinear dynamics.

3.2. **Vectored thrust propulsion.** As part of an experimental project funded by the HARV project at NASA's Dryden Flight Research Center, we have recently undertaken a study of the problem of robust control of vectored-thrust, high-performance jet aircraft. Very few design methods are available for building robust control laws for this type of system, especially when it is to be used for aggressive maneuvers far away from an equilibrium point. We are currently concentrating on the control of a small experimental system powered by a ducted fan engine, transitioning between hover and forward flight.

The system which we have built is shown in Figure 3. It consists of a high-efficiency electric motor with a 6-inch diameter blade, capable of generating up to 9 Newtons of thrust. Flaps on the fan allow the thrust to be vectored from side to side and even reversed. The engine is mounted on a three degree of freedom stand which allows horizontal and vertical translation as well as unrestricted pitch angle. See [5] for a detailed description.

In terms of trajectory tracking, our goal is to allow real-time input of the desired xy velocity of the fan and to design controllers that can track any trajectory within the performance limits of the fan and that respond well to commands outside of those limits. Standard approaches to this problem, such as I/O linearization, cannot be used here for several reasons: 1) the system has non-minimum phase zero dynamics with respect to the xy position, 2) the system is not full state linearizable with static feedback (nor obvious dynamic feedbacks), and 3) we do not have access to higher derivatives of the desired trajectory. For these reasons we must study the application of more advanced geometric techniques to generate controllers for the system.

The equations of motion for the full ducted fan system do not satisfy any of the currently known conditions for differential flatness. However, one interesting feature of the system is that the dynamics have a symmetry corresponding to the xy position of the fan. By reducing the dynamics of the system by this symmetry group, we can lower the dimension of the equations needed to describe its motion and then reconstruct the full trajectory given the reduced trajectory. Our initial calculations show that reduced dynamics *are* differentially flat on an open dense set of the reduced space, and so we can completely characterize the solution curves for the system on that set. Unfortunately, the points which are not included correspond to the pitch velocity of the fan being zero, and hence there is a singularity which occurs whenever the pitch velocity changes sign. Typically, this occurs very often in routine maneuvers.

A more promising approach in this example may be to approximate the full system by a simpler one which is differentially flat. Based on recent work by Martin et al [20], it is possible to show that a planar version of the ducted fan with no stand dynamics is flat using the center of oscillation of the system. This indicates that it may be possible to find a simple version of the stand dynamics which would preserve flatness and allow approximate trajectory generation. We plan to pursue this avenue of investigation as part of our work on this project.

Although this experiment is much simpler than a real vectored-thrust aircraft, it contains many of these same basic features and has the advantage that aggressive control laws can be applied and tested much more easily and safely. By using this experiment to test out new theory, we hope to speed the transfer of that theory into applications such as the HARV project and other highly maneuverable aircraft systems.

3.3. **Other applications.** The applications listed above are two in which we already have access to models and experimental systems. However, there are many other systems in which the theory we plan to explore and software we plan to develop can be used.

One potential area of future application is autonomous underwater vehicles (AUVs). Dynamically, these system share many of the properties of flight control systems with the exceptions that buoyancy cannot be ignored and drag becomes significant at much lower velocities. From a controls point of view, the tasks to be achieved are very similar: trajectory generation and tracking are major issues. One difference is that AUVs often operate near zero velocity, for example when being used for inspection purposes. In such cases, it turns out that the linearization of the system is not controllable and hence strongly nonlinear techniques, similar to those developed for mechanical systems with nonholonomic constraints, must be employed [8].

Other sources of nonlinear mechanical systems include multi-fingered robot hands and land-based locomotion systems. In both of these cases the constraints between the mechanism and its environment are needed to determine the dynamics and the recent results of Bloch et al. [3] give a detailed description of the geometric structure of the dynamics.

4. Summary of Prior Research Accomplishments

My past and current research interests are in the broad area of nonlinear control of mechanical systems. I have worked on problems in robotic manipulation, nonholonomic motion planning, nonlinear control theory, and geometric mechanics, at both the theoretical and applied levels.

My work in multifingered hands has included experimental comparisons between different controls and control architectures [11, 27, 37], as well as more theoretical research aimed at providing a unified and structured view of dextrous manipulation [25, 6, 28]. Other related work includes analysis and control techniques for complex actuator networks [6], and analysis and control of multifingered hands with flexible links [37]. Much of my work in this area is summarized in a recent graduate level textbook for which I was the lead author [26]. This book expands on the work in my PhD thesis and provides a unique perspective on the study of robot kinematics, dynamics, and control which makes use of modern screw theory to exploit the geometry of rigid motion in $\mathbb{R}^3$. It is currently being used as a textbook at several universities around the country.

The bulk of my research group's work over the past three years at Caltech has been in the analysis and control of mechanical systems with nonholonomic constraints. My coworkers and I have made fundamental contributions in analyzing the geometric structure of nonholonomic control systems and in utilizing this structure to solve problems in motion planning [29, 18, 40], trajectory tracking [30, 42], and feedback stabilization [38, 22, 21]. We introduced the notion of "chained" and "power" forms for nonholonomic systems [29, 24, 38], which are now standard tools used by other researchers in the field. Our results in this area have yielded new theoretical insights into the geometric structure of more general nonlinear systems (see, for example, [23]) and have also lead to fundamental new results in geometric mechanics [16, 3].

A more recent area of interest is the use of exterior differential systems for modeling, analyzing, and synthesizing nonlinear control systems. Preliminary results developed by my research group over the past year have already generated new insights into trajectory generation for exterior differential systems which have direct, practical relevance to the nonlinear control applications [41]. This promises to be an exciting new area which will help overcome some of the current limitations of nonlinear control theory.

Effective research in nonlinear control requires both sound theoretical methods and experimental verification. To test out new techniques and motivate future theoretical advances, my students and I have set up a modern controls laboratory and developed several experiments for use in our research. These experiments include a small nonholonomic mobile robot [21] and a planar two-fingered hand with flexible links and tendons [37]. We have also developed a real-time kernel [31] to support our experimental activities and maintain a cluster of ten engineering workstations and five 486-based personal computers for use in our research.

The most advanced mechanical systems experiment in our lab is the ducted fan system described earlier. It consists of a small, three degree of freedom, ducted fan engine which transitions between hover and forward flight using vectored thrust. Past work has included design, construction, and modeling [5] as well as an initial design study comparing existing linear and nonlinear design methodologies [15]. The ducted fan system is strongly nonlinear and highly uncertain, allowing us to test our design methods on realistic applications which have many of the features and limitations of industrial control systems. It will play a central role in design and verification of the advanced techniques that will be developed as part of this proposal.

5. Work Plan

Work to be performed under this proposal is divided into four broad areas:

- Computational methods for real-time trajectory generation for nonlinear, mechanical systems
- Theoretical and computational methods for analyzing the robustness to perturbations which preserve the mechanical nature of the system
- Theoretical and experimental studies to explore the trade-offs between aggressiveness in trajectory generation and robust performance
- Implementation of proposed techniques in industrial applications

We now briefly describe our goals in each of these areas.

The first problem which we plan to study is real-time generation of trajectories for nonlinear systems. By initially concentrating on differentially flat systems, we will generate software tools which will allow online computation of feasible state-space trajectories given a desired task. Examples include transition between relative equilibria for the towed cable system and aggressive maneuvering for the ducted fan (and HARV) systems. Theoretical work will be directed at deriving necessary and sufficient conditions for differential flatness of Lagrangian systems. Future work will be dictated by our experience with examples, but might include, for example, generation of an approximate theory for differential flatness (since this appears useful in the ducted fan example and perhaps other flight control systems).

In conjunction with the work on trajectory generation, we plan to begin a detailed study of robust tracking issues for mechanical systems. Although well-developed in the linear literature, robustness of nonlinear systems has in most instances relied on the use of special normal forms or restrictive classes of model perturbations. The notion of robust *performance* (rather than stability) is also of great importance and has largely been unstudied in the nonlinear literature. By focusing on mechanical systems, we hope to exploit the second order and geometric nature of the dynamics to obtain better results. We will also explore the use of computational techniques (along the lines of initial work described in [39]) to compute robustness and performance along trajectories.

The work on trajectory generation and robust tracking will be combined by performing various trade-off studies on the ducted fan experiment and other physical systems to gain insight into the interaction between the feedforward and feedback phases. In particular, we plan to look at the use of cost functions for trajectory generation which depend on the controllability of the system in a given region of the operating envelope and the effects on performance of using only approximately feasible trajectories (due to modeling errors).

Finally, a major goal of this project is implementation of these techniques in industrial and military applications. By making contact with industrial applications, we hope to guide our research towards problems of practical importance. The author currently has a working relationship with members of the ROSS project at Northrop Grumman and has contacts with Dryden Flight Research Center (where the HARV project is based). These are described in more detail in the next section.

6. Collaborations with Industry and Government Labs

As part of this proposal we plan to develop collaborations with relevant industries to help guide the research effort and also transfer results of our research into commercial and military applications.

Contact with the ROSS project at Northrop Grumman Corporation has already been made and a letter of support is included Section I at the end of this proposal. We expect this to be a very rich collaboration since the success of the ROSS project will in part depend on finding good solutions to the types of nonlinear problems considered here. The nature of the collaboration will probably be through interaction between the PI and Northrop Grumman personnel as well sharing of simulation models and experimental data.

As part of a joint project with J. Doyle funded by the HARV project at NASA Dryden Flight Research Center, the PI has already been involved in research related to control of vectored thrust aircraft. The work funded by NASA has concentrated on development of the ducted fan experiment and the use of robust linear control combined with existing nonlinear methods. It complements the nonlinear controls research described in this proposal. The collaboration with Dryden is expected to continue and would grow substantially as the result of the work described in this proposal.

Finally, the PI has recently contacted personnel at Honeywell and their appears to be a natural connection with the work described here and current and future work being undertaken at Honeywell (in part, related to the HARV project). Increased collaboration with Honeywell and other industries will be pursued as a natural part of the work proposed here.

7. Impact of Proposed Research

This work is very much motivated by existing and future applications in flight control and robotics. Several of the specific problems which are driving the need for new results are being pursued in cooperation with industry and government labs, allowing for direct transfer of these results into applications.

At the theoretical end, problems in control theory are already driving many new results in geometric mechanics and dynamical systems theory. The research to be pursued here will help focus attention on problems of practical importance which require a more complete understanding of the mathematical structure of mechanical systems. This synergy between mathematics, control theory, and applications has the potential to allow substantial gains in all three areas and would greatly increase our ability to design, build, and control the increasingly complex systems that are required if the US is to remain a leader in science and technology.

Teaching Plan

1. Introduction

Caltech is a small, research oriented university which focuses almost exclusively on studies in science and enginnering. At the undergraduate level, Caltech enrolls approximately 900 undergraduates, of which about 200 are graduated each year. A high fraction of these students, typically around 60%, go on to graduate school. Within Mechanical Engineering, students can take courses in mechanical systems and engineering design, mechanics, thermal systems and applied fluid mechanics. A typical program consists of courses in applied mathematics, statics and dynamics, fluids, solids, thermodynamics, and mechanical systems. In addition, Institute requirements include 2 years each of math and physics, 1 year of chemistry, and 4 years of humanities and social sciences.

Because of its small size and research orientation, Caltech is an ideal place for developing new courses and trying new teaching techniques. Classes are small, the students are extremely bright, and Institute resources for use in teaching are usually available with minimal effort. Past examples of courses and textbooks developed at Caltech include the calculus books by Apostol, used for freshman and sophomore math, and the *Mechanical Universe* series for teaching physics, developed jointly by the physics and math departments.

Undergraduate research is extremely common at Caltech. In a recent survey for a student/faculty conference, about 70% of graduating seniors said that they had done some type of research during their time at Caltech and a roughly equal number of incoming freshman cited it as one of the reasons for coming to Caltech. The Summer Undergraduate Research Fellowship (SURF) program allows students to get paid for research during the summer and many departments offer a senior thesis option.

At the graduate level, there are about 1,100 students enrolled at Caltech, almost all pursuing a PhD degree. In addition to traditional engineering disciplines, Caltech also has two interdisciplinary PhD options: Computation and Neural Systems (CNS) and Control and Dynamical Systems (CDS). The CDS department was established in 1994 and currently consists of four primary faculty: Doyle, Morari, Murray, Wiggins (most departments are quite small; Mechanical Engineering has 9 faculty). The CDS program is described in more detail below.

2. Teaching Plan

My teaching plan is very much shaped by the style of education at Caltech and the particular resources which are available to me here. I hold a joint appointment in Mechanical Engineering and Control and Dynamical Systems (CDS) and have been very active in undergraduate issues. My broad teaching goals are:

1. to continue development of a new curriculum in control and dynamical systems which can be used as a model for similar programs elsewhere;
2. to continue building experimental laboratories for graduate and undergraduate teaching; and
3. to search for new ways of promoting undergraduate research at Caltech.

Details of my planned activities in these areas are given below.

2.1. **Control and Dynamical Systems.** January 1994 marked the start of a new PhD program at Caltech in Control and Dynamical Systems (CDS). I played a very active role in the development of this program and plan to continue to help shape the curriculum into a novel program of study which exploits the use of powerful computational techniques and advanced mathematics to analyze and solve problems in the control of highly complex, dynamical systems. This is a graduate-only option, however we do teach a one year course at the undergraduate level which is also taken by some graduate students from other departments.

As part of the CDS program, I am currently developing and updating courses in control theory, applied mathematics, and experimental methods. Over the next five years, I plan to teach existing undergraduate and graduate courses in control theory and robotics, as well as to continue developing new courses at the senior and graduate level in control technology and nonlinear control theory.

2.2. **Experimental facilities for instruction.** Professors Morari, Doyle, and I have received an NSF grant as part of the combined research and curriculum development program to build a modular curriculum in Control Systems that can be flexibly integrated into undergraduate and graduate programs at Caltech and other universities. A key component of the curriculum is a set of experiments which range from teaching basics to providing benchmarks that challenge new design methodologies. To realize our objectives, we are developing an integrated infrastructure that takes advantage of the latest developments in hardware and software to provide students and researchers with a unified framework for computation, simulation, and experimentation.

I am the person responsible for coordinating most of the experimental aspects of this proposal. Much of the effort is in developing the infrastructure needed to perform experimental work with low overhead. We are developing a real-time software package which runs on 486-based personal computers and can be used for experiments ranging from simple proportional derivative controllers balancing inverted pendulums to complex nonlinear controllers used to control the ducted fan experiment described above. We have made substantial progress thus far and I plan to concentrate future efforts on making the software more robust and releasing it to others.

I am also supervising the development of two new experiments for use in our courses. The first is a modified version of the ducted fan which can be more easily replicated by other universities. The second is a model heating, ventilation, and air condition (HVAC) system which will have multiple rooms that must be maintained at different temperatures and comfortable humidity levels over a wide range of operating conditions and in presence of unknown disturbances.

My motivation in building these experiments is twofold. On the one hand, these experiments are very much needed in order to educate students in both the practical and theoretical aspects of control (and the interactions between them). However, these experiments also serve as a training ground for students who will eventually do research in my group. By having a consistent hardware and software interface, the amount of effort needed to incorporate experimental results into research is drastically reduced. By making this infrastructure available to others we hope to help promote the use of experiments at other universities as well.

2.3. **Undergraduate research.** Because of the nature of undergraduates who come to Caltech, there is a high degree of interest on the part of undergraduates in pursuing research. I have sponsored a number of student projects and am looking for ways to encourage more students and faculty in Mechanical Engineering to get involved in projects. As part of my activities in setting up experimental facilities, I have already made some space available for independent undergraduate projects in a lab equipped with a full complement of computers and test equipment. This is primarily for the use of faculty who don't have lab space available for undergraduate research.

One of the typical avenues of research for undergraduates is the Summer Undergraduate Research Fellowship (SURF) program sponsored by Caltech. SURF projects consist of a 10 week project in which a student works with a faculty member on a problem of mutual interest. In practice, this type of research is often not well-suited for engineers since it is difficult to find a 10 week project which is not simply some small subset of a larger effort. Working on small parts of bigger projects tends not to be very exciting and often students do not experience the excitement of doing research that comes with a larger (and more time-consuming) project.

I am interested in exploring new ways of getting students involved with research, possibly separate from the SURF program. I have helped set up a senior thesis course in Mechanical Engineering and, along with several of my colleagues, I hope to establish an option whereby a student can graduate with a BS in Mechanical Engineering "with thesis". The topic of undergraduate thesis would be approved by the faculty and would be read and signed by a thesis committee of two to three professors.

3. Summary of previous accomplishments

In the three years that I have been at Caltech, I have been involved in a number of different activities which are consistent with the goals that I have outlined above. I have taught classes in kinematics, robotics, experimental methods in mechanical engineering, linear control theory, applied mathematics, and nonlinear control theory. I have also written a textbook [26] which is used for teaching the mixed undergraduate/graduate class in kinematics and robotics.

One of my main activities has been to improve the laboratory facilities available for instructional use. I have been in charge of setting up a new mechanical systems laboratory which is used for courses in both mechanical enginnering and CDS, and have helped design and build two new experiments for use in that lab: a rotary, double inverted pendulum and an axial flow compressor which exhibits rotating stall and surge. Two other experiments are underway as part of the NSF-sponsored grant for curriculum development described above. I also taught the undergraduate experimental methods course last year and began updating some of the labs in that course.

One of the most pleasant aspects of Caltech is getting involved with undergraduates who are interested in research. I have sponsored several undergraduate research projects, both as part of the Summer Undergraduate Research Fellowship (SURF) program and as senior thesis projects. I have had a total of 5 SURF students and have sponsored 3 senior theses. Two of the senior thesis projects have resulted in conference papers: one describing the design and construction of the ducted fan engine described earlier [5] and the other on the use of air injection as an actuator for controlling instabilities in axial flow compressors [17].

Undergraduates who work in my lab are treated as graduate students and I feel that the undergraduates, my graduates students, and I all find undergraduate research an extremely valuable experience. As I described above, I plan to continue to search for new ways to encourage it.

Another of my major activities over the past several years has been in developing the curriculum for the new Control and Dynamical Systems (CDS) option at Caltech. I have developed two new courses for that option: one in differential geometry (manifolds, vector fields, Lie groups) and the other in differential geometric control theory (controllability, feedback linearization, etc). I am currently involved in developing courses and experiments for the curriculum development effort sponsored by NSF and described in the previous section.

At the campus level, I recently served on an elected faculty committee to re-evaluate the core curriculum at Caltech. This role provided me with an opportunity to help direct the future of the undergraduate educational experience at Caltech and, in so doing, to help me develop new ideas for how best to teach science and engineering to undergraduates. I was very active on the committee and help draft a large part of the final report from the committee. The recommendations of that committee resulted in the formation of a task force to make specific and significant changes to the core curriculum, the first in perhaps 25 years. We hope to see these changes implemented in the next few years.

Departmental Endorsement

Richard M. Murray was appointed as an Assistant Professor of Mechanical Engineering at Caltech on September 2, 1991. This was his first full-time tenure-track appointment.

I have read and endorse the Career Development Plan.

____

John Seinfeld, Chairman
Division of Engineering and Applied Science

Bibliography

1. R. Abraham and J. E. Marsden. *Foundations of Mechanics.* Benjamin/Cummings, second edition, 1985.
2. V. I. Arnold. *Mathematical Methods of Classical Mechanics.* Springer-Verlag, second edition, 1989.
3. A. M. Bloch, P. S. Krishnaprasad, J. E. Marsden, and R. M. Murray. Nonholonomic mechanical systems with symmetry. Technical Report CIT/CDS 94-013, California Institute of Technology, 1994. (Submitted to *Archive for Rational Mechanics.* Available electronically via `http://avalon.caltech.edu/cds/`).
4. D. J. Bugahski and D. F. Enns. Nonlinear control law with application to high angle-of-attack flight. *Journal of Guidance, Control, and Dynamics*, 15(3):761–767, 1992.
5. H. Choi, P. Sturdza, and R. M. Murray. Design and construction of a small ducted fan engine for nonlinear control experiments. In *Proc. American Control Conference*, pages 2618–2622, 1994.
6. D. C. Deno, R. M. Murray, K. S. J. Pister, and S. S. Sastry. Finger-like biomechanical robots. In *Proc. IEEE International Conference on Robotics and Automation*, pages 566–572, 1992.
7. M. A. Dornheim. X-31, F-16 MATV, F/A-18 HARV explore diverse missions. *Aviation Week and Space Technology*, 140(16):46–47, 1994.
8. O. Egeland and E. Berglund. Control of an underwater vehicle with nonholonomic acceleration constraints. In *Proc. IFAC Symposium on Robot Control (SYROCO),*, pages 959–964, 1994.
9. M. Fliess, J. Levine, P. Martin, and P. Rouchon. On differentially flat nonlinear systems. *Comptes Rendus des Séances de l'Académie des Sciences*, 315:619–624, 1992. Serie I.
10. M. Fliess, J. Levine, P. Martin, and P. Rouchon. Linéarisation par bouclage dynamique et transformations de Lie-Bäcklund. *Comptes Rendus des Séances de l'Académie des Sciences*, 317(10):981–986, 1993. Serie I.
11. K. Hollerbach, R. M. Murray, and S. S. Sastry. An experimental study of hierarchical control laws for grasping and manipulation using a two-fingered planar hand. In *Proc. IEEE International Conference on Robotics and Automation*, pages 2770–2775, 1992.
12. L. R. Hunt, R. Su, and G. Meyer. Global transformations of nonlinear systems. *IEEE Transactions on Automatic Control*, AC-28(1):24–31, 1983.
13. A. Isidori and C. I. Byrnes. Output regulation of nonlinear systems. *IEEE Transactions on Automatic Control*, 35(2):131–140, 1990.
14. B. Jakubczyk and W. Respondek. On linearization of control systems. *Bulletin de L'Academie Polonaise des Sciences, Série des sciences mathématiques*, XXVIII:517–522, 1980.
15. M. Kantner, B. Bodenheimer, P. Bendotti, and R. M. Murray. An experimental comparison of controllers for a vectored thrust, ducted fan engine. In *Proc. American Control Conference*, 1995. (submitted).
16. S. D. Kelly and R. M. Murray. Geometric phases and robotic locomotion. Technical Report CIT/CDS 94-014, California Institute of Technology, September 1994. (submitted to *J. Robotic Systems*).

17. A. Khalak and R. M. Murray. Experimental evaluation of air injection for actuation of rotating stall in a low speed, axial fan. In *Proc. ASME Turbo Expo*, 1995. (submitted).
18. J-P. Laumond, P. Jacobs, M. Taix, and R. M. Murray. A motion planner for nonholonomic mobile robots. *IEEE Transactions on Robotics and Automation*, 1994. (to appear).
19. J. E. Marsden. *Lectures on Mechanics.* LMS Lecture Note Series. Cambridge University Press, 1991.
20. P. Martin, S. Devasia, and B. Paden. A different look at output tracking: control of a vtol aircraft. In *Proc. IEEE Control and Decision Conference*, 1994. (to appear).
21. R. T. M'Closkey and R. M. Murray. Convergence rates for nonholonomic systems in power form. In *Proc. American Control Conference*, 1993.
22. R. T. M'Closkey and R. M. Murray. Exponential stabilization of driftless nonlinear control systems via time-varying, homogeneous feedback. In *Proc. IEEE Control and Decision Conference*, 1994. to appear.
23. R. M. Murray. Applications and extensions of Goursat normal form to control of nonlinear systems. In *Proc. IEEE Control and Decision Conference*, pages 3425–3430, 1993.
24. R. M. Murray. Nilpotent bases for a class of non-integrable distributions with applications to trajectory generation for nonholonomic systems. *Mathematics of Control, Signals, and Systems*, 1994. (in press).
25. R. M. Murray, D. C. Deno, K. S. J. Pister, and S. S. Sastry. Control primitives for robot systems. *IEEE Transactions on Systems, Man and Cybernetics*, 22(1):183–193, 1992.
26. R. M. Murray, Z. Li, and S. S. Sastry. *A Mathematical Introduction to Robotic Manipulation.* CRC Press, 1994.
27. R. M. Murray and S. S. Sastry. Control experiments in planar manipulation and grasping. In *Proc. IEEE International Conference on Robotics and Automation*, pages 624–629, 1989.
28. R. M. Murray and S. S. Sastry. Grasping and manipulation using multifingered robot hands. In R. W. Brockett, editor, *Robotics: Proceedings of Symposia in Applied Mathematics, Volume 41*, pages 91–128. American Mathematical Society, 1990.
29. R. M. Murray and S. S. Sastry. Nonholonomic motion planning: Steering using sinusoids. *IEEE Transactions on Automatic Control*, 38(5):700–716, 1993.
30. R. M. Murray, G. Walsh, and S. S. Sastry. Stabilization and tracking for nonholonomic systems using time-varying state feedback. In *IFAC Symposium on Nonlinear Control Systems Design (NOLCOS)*, pages 182–187, Bordeaux, France, 1992.
31. R. M. Murray and E. L. Wemhoff. *Sparrow 2.0 Reference Manual.* California Institute of Technology, 1994.
32. A. Packard. Gain scheduling via linear fractional transformations. *Systems and Control Letters*, 22(2):79–92, 1994.
33. A. Packard, J. Doyle, and G. Balas. Linear, multivariable robust-control with a μ-perspective. *ASME Journal of Dynamic Systems, Measurement and Control*, 115(2B):426–438, 1993.
34. J. J. Russell and W. J. Anderson. Equilibrium and stability of a circularly towed cable subject to aerodynamic drag. *Journal of Aircraft*, 14(7):680–686, 1977.
35. J. S. Shamma. Robust stability with time-varying structured uncertainty. *IEEE Transactions on Automatic Control*, 39(4):714–724, 1994.

36. R. A. Skop and Y-I. Choo. The configuration of a cable towed in a circular path. *Journal of Aircraft*, 8(11):856–862, 1971.
37. S. Sur, R. Behnken, and R. M. Murray. Grasping with flexible link fingers: An initial study. In *Proc. IFAC Symposium on Robot Control (SYROCO),*, 1994.
38. A. Teel, R. M. Murray, and G. Walsh. Nonholonomic control systems: From steering to stabilization with sinusoids. *International Journal of Control*, 1994. (to appear).
39. J. Tierno, R. M. Murray, and J. C. Doyle. An efficient algorithm for performance analysis of nonlinear control systems. In *Proc. American Control Conference*, 1995. (submitted).
40. D. Tilbury, R. M. Murray, and S. S. Sastry. Trajectory generation for the N-trailer problem using Goursat normal form. *IEEE Transactions on Automatic Control*, 1994. (accepted for publication).
41. M. van Nieuwstadt, M. Rathinam, and R. M. Murray. Differential flatness and absolute equivalence. Technical Report CIT/CDS 94-006, California Institute of Technology, 1994. (Submitted to *SIAM J. Control and Optimization*).
42. G. Walsh, D. Tilbury, S. Sastry, R. Murray, and J-P. Laumond. Stabilization of trajectories for systems with nonholonomic constraints. *IEEE Transactions on Automatic Control*, 39(1):216–222, 1994.

Budget Justification

Funds are requested to provide support for two graduate students, partial support for travel to one domestic conference per year, and funds for materials and expenses. The requested funds for supplies and expense were determined based on average costs on previous projects of similar nature. Additional funds for graduate students in the form of fellowships and teaching assistantships will be sought in order to augment the number of students working on this proposal.

Institute policy is to provide each graduate student employee who meets a required average work week with tuition and fees. A portion of this cost is requested as a benefit (exempt from indirect costs) equivalent to 80% of the graduate research assistant salary effective October 1, 1997.

Students will use computers and experimental facilities purchased with funds from other sources. Software and other computer support costs will be paid for with funds available to the PI from other sources.