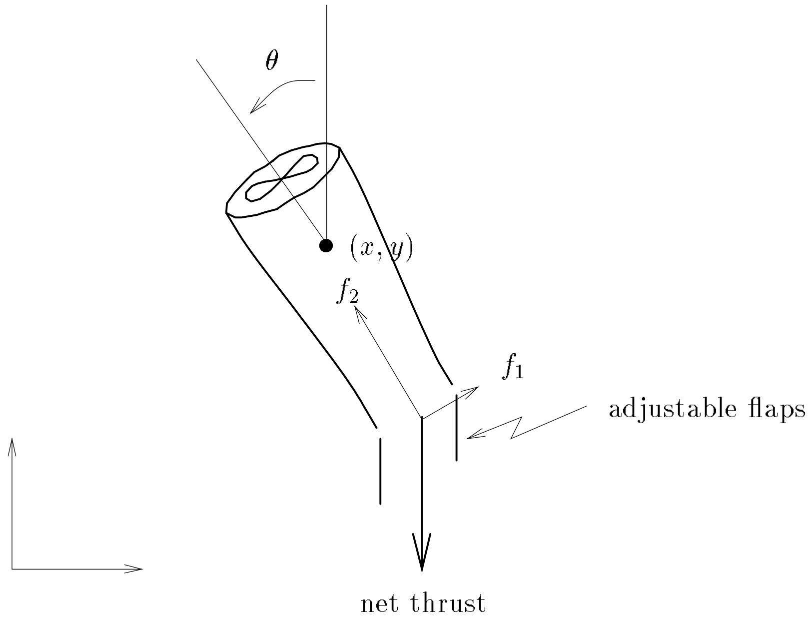
CALIFORNIA INSTITUTE OF TECHNOLOGY
Control and Dynamical Systems

CDS 111
Problem Set #1

R. Murray | Issued: 1 Apr 96
Spring 1996 | Due: 10 Apr 95

The purpose of this problem set is to remind you of the concepts that you should be familiar with from your undergraduate course in controls and also to introduce you to the MATLAB software package.

Consider the motion of a small model aircraft powered by a vectored thrust engine, as shown below.



Let (x, y, θ) denote the position and orientation of the center of mass of the fan. We assume that the forces acting on the fan consist of a force f_1 perpendicular to the axis of the fan acting at a distance r and a force f_2 parallel to the axis of the fan. Let m be the mass of the fan, J the moment of inertia, g the gravitational constant, and D the damping coefficient. Then the equations of motion for the fan are given by:

$$\begin{aligned} m\ddot{x} &= f_1 \cos\theta - f_2 \sin\theta - d\dot{x} \\ m\ddot{y} &= f_1 \sin\theta + f_2 \cos\theta - mg - d\dot{y} \\ J\ddot{\theta} &= r f_1. \end{aligned}$$

It is convenient to redefine the inputs so that the origin is an equilibrium point of the system with zero input. If we let $u_1 = f_1$ and $u_2 = f_2 - mg$ then the equations become

$$\begin{aligned} m\ddot{x} &= -mg\sin\theta - d\dot{x} + u_1\cos\theta - u_2\sin\theta \\ m\ddot{y} &= mg(\cos\theta - 1) - d\dot{y} + u_1\sin\theta + u_2\cos\theta \\ J\ddot{\theta} &= r u_1. \end{aligned} \tag{1}$$

These equations are referred to as the *planar ducted fan equations.*

1. If we linearize these equations around the hover position ($\theta = 0$), we can obtain a two input, two output linear system with

$$u = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \qquad z = \begin{bmatrix} x \\ y \end{bmatrix}$$

   Write down the transfer function matrix $H_{zu}(s)$ between the inputs and the outputs. (Your answer should be a diagonal matrix of transfer functions.)

For the remainder of the problems in this homework set, consider only the transfer function $P(s) := H_{z_2 u_2}$ between u_2 and the y position of the planar ducted fan. Use the following values for the parameters of the system:

$$\begin{array}{lll} g = 0.52 \text{ m/sec}^2 & m = 4.25 \text{ kg} & \\ & & d = 0.1 \text{ kg/sec} \\ r = 26 \text{ cm} & J = 0.0475 \text{ kg m}^2 & \end{array}$$

These are roughly the values for the planar ducted fan in 08 Thomas, so you should be able to judge whether the performance of your controller is reasonable. The reason that gravity is not 9.8 m/sec^2 is because of the presence of a counterweight to offset the weight of the fan.

2. Plot a pole-zero diagram for $P(s)$. Is the open loop system stable?

3. Plot the bode plot of $P(s)$. Is the closed loop system with unity gain feedback stable?

4. Design a controller which stabilizes the system about $\theta = 0$ and tracks a command signal for the height y. Plot the loop transfer function for your controller and compute the gain and phase margin.

   Hint: try a compensator of the form

$$C(s) = k\tfrac{s+a}{s+b} \qquad b \gg a.$$

   You should be able to pick reasonable values of a, b, and k by looking at the open loop Bode plot or using a root locus diagram.

5. For the compensator that you designed in Exercise 3, plot the Nyquist plot of the open loop transfer function and verify the gain and phase margin that you computed.

6. Plot a root locus diagram for your controller with respect to increasing gain. Is your controller stable for all positive gains?

7. Plot the step response for the system and compute the settling time and overshoot for your controller.

8. For the real ducted fan system there are many nonlinear and unmodeled dynamics which enter the system. One source of unmodeled dynamics in our existing experimental system is that the motor controller for the fan is not perfect and there is a 250 msec lag between the commanded thrust and the actual thrust. Determine if your controller is robust to a 250 msec actuator delay by using a Bode plot or Nyquist plot to compute stability of the modified closed loop plant. (Do *not* update your controller design, just modify the plant.)

9. Plot a root locus diagram for your controller in the presence of the time lag in the previous exercise. Is your controller stable for all positive gains?