AFOSR-PRET Grant F49620-95-1-0419

Robust Nonlinear Control Theory with Applications to Aerospace Vehicles

Richard Murray
Control and Dynamical Systems
California Institute of Technology
Pasadena, CA 91125

John Doyle, Jerry Marsden, Stephen Wiggins
Caltech Control and Dynamical Systems

Gary Balas
University of Minnesota

Blaise Morton
Honeywell

Progress Report
1 August 1996 to 31 July 1997

1 Status of Effort

Research conducted under this proposal has continued to make strong progress in the area of nonlinear control of mechanical systems, robustness analysis for nonlinear systems, real-time trajectory generation, and linear parameter varying (LPV) control. An increased emphasis is being placed on computational methods in nonlinear control as well as fundamental theory of nonlinear control of mechanical systems. A series of invited sessions and workshops have been used to communicate the results of this research effort to the controls community, in addition to implementation and testing of algorithms on university experiments and pilot-in-the-loop simulations (performed by Balas).

2 Accomplishments

Trajectory Generation and Tracking Using Differential Flatness

Murray and his coworkers have been investigating techniques for online trajectory generation for flight control applications, motivated by problems in uninhabited aerial vehicles. The role of trajectory generation is important in these systems since the pilot is not in the plane and therefore cannot provide feasible guidance commands in highly aggressive flight situations. We are investigating techniques for generating state and input trajectories which satisfy the equations of motion and trade off tracking performance for internal stability. Currently we are not explicitly considering actuator constraints (magnitude and rate limits), although we believe that the theory can be extended to handle this case along the lines of the techniques proposed here.

The results to date in this area include the development of real-time algorithms for trajectory generation and application of those algorithms to a small flight control experiment at Caltech. Results on the experiment demonstrate the value of performing real-time trajectory generation for situations in which the desired vehicle position is specified in real time (via a joystick) and there is a clear increase in performance over standard linear techniques.

We have also derived conditions for checking differential flatness for different classes of nonlinear mechanical systems. In particular, for simple mechanical systems (kinetic plus potential energy) with one fewer input that configuration variable, we have given necessary and sufficient conditions for a system to be flat in terms of the covariant derivatives of certain objects determined from the way in which external forces enter the system.

Numerical Algorithms for Analysis and Design of Nonlinear Control Systems

We continue to develop software tools for robustness analysis of nonlinear systems. Our original work in this area (with J. Tierno, now at Honeywell) considered the problem of worst case analysis of a nonlinear system along a give trajectory. Although local in nature, the techniques provided the ability to incorporate noise, uncertainty in initial conditions, (real) parametric uncertainty, and a class of unmodeled dynamics. The information provided by this numerical tool complements the more traditional Monte Carlo approach for analyzing robust performance of a system.

In the past 24 months, we have developed some efficient algorithms for verifying the stability of uncertain discrete time piecewise linear systems. While piecewise linear systems are intuitively simple, they are computationally hard. Two approaches to verifying stability are presented. For each approach, separate necessary and sufficient conditions are given. The first approach requires the solution of a linear matrix inequality. This method is only applicable to a restricted class of piecewise linear systems, and is generally very conservative. It is demonstrated that for most piecewise linear systems, these conditions yield no information. The second, more general, approach is based upon robust simulation. This method is useful for all piecewise linear systems, and will always yield a definitive answer. If a system initially satisfies necessity, but fails sufficiency, these algorithms can be systematically refined and after a finite number of refinements, a definitive answer is guaranteed.

In the past 6 months, we have begun to develop new software for design of nonlinear control systems. This software represents a nonlinear system by a collection of Taylor series representations of arbitrary order at a given set of points. By using multiple local approximations, we are able to better represent nonlinear function in a way which is computationally tractable and fits with current nonlinear identification techniques (which include table lookups and linearizations about operating points). An initial library of routines has been created which allows standard operations such as Lie brackets and involutivity checks. Future work will concentrate on computation of (approximately) flat outputs for nonlinear systems.

Feedback Stabilization of Mechanical Systems Using Controlled Lagrangians

In joint work with Tony Bloch and Naomi Leonard, Marsden is continuing to investigate a structured approach to stabilization of Lagrangian systems with symmetry. The strategy is to write the controlled system as the Euler-Lagrange equations for a modified Lagrangian called the *controlled Lagrangian.* The Euler-Lagrange equations for the controlled Lagrangian describe a feedback system where the new terms introduced by the modifications of the Lagrangian are identified with control forces. Since the controlled system is Lagrangian by construction, energy methods can be used to find control gains that yield closed-loop stability.

The procedure is demonstrated for the problem of stabilization of an inverted pendulum on a cart and for the problem of stabilization of rotation of a rigid spacecraft about its unstable intermediate axis using a single internal rotor. Similar results hold for the dynamics of an underwater vehicle with internal rotors as well as the inverted spherical pendulum on a two dimensional moving platform with a spherical joint.

Progress has been made on the general theory for when such control systems can be realized this way (the matching problem), when they yield stabilization and what the region of attraction is when dissipative controls are added. We expect to write up a longer version of our research on this in the coming months. A preprint outlining the basics of our ideas will appear in the 1997 CDC in San Diego.

Linear Parameter Varying Control and Its Applications

The past year our research at the University of Minnesota under the AFOSR PRET program has focused on applying linear-parameter varying (LPV) H_∞ control design methods to aerospace vehicles. We have developed heuristic approaches, similar to D-K iteration, to include performance and robustness objectives in the linear-parameter varying framework. This approach has been applied successfully to the design of controllers for the F-14 lateral-directional axis during powered approach. These controllers are designed to operate in the powered-approach flight envelope between $\alpha = 2$ degrees, 180 knots to $\alpha = 14$ degrees and 128 knots based on four linearized models at $\alpha = 2, 6, 10.5$, and 14 degrees angle-of-attack. The LPV controllers, which schedule on angle-of-attack, were implemented successfully in the full order F-14 simulation and tested in pilot-in-the-loop simulations at the Naval Air Warfare Center in Patuxent River, Maryland. We believe this is the "first" pilot testing of controllers designed using LPV control design techniques.

Based on the success of this work, these techniques are currently being applied to design autopilots for a high performance, highly coupled missile. As part of our outreach mission, a two-day workshop at the 1997 American Control Conference entitled "Theory and Application of Linear Parameter Varying Control Techniques" is being offered by Andy Packard and myself to expose attendees to the latest results in parameter-dependent control and their application to real world control problems. We will provide attendees with a brief background on robust control, linear matrix inequalities (LMIs), and solutions of LMIs using convex optimization techniques. The underlining theory for LPV systems is presented as well as the solution to several gain-scheduled control problems that arise in the LPV framework. Application of these methods to the F-14 lateral-directional axis powered-approach flight control system, design of missile autopilots and process control examples are presented. We offered a similar two-day short course to 6 engineers at Raytheon and the Johns Hopkins University Applied Physics Laboratory in January, 1997.

In addition to our work on linear-parameter varying control designs, we have designed and are developing an actuator for structural vibration attenuation using shape memory alloys (SMAs). Our past research has shown that SMAs can be successfully used as passive vibration attenuation devices. To extend this work, we have design an active SMA actuator for vibration attenuation. We are able to achieve a bandwidth of approximately 10 Hz, though the force levels are very small (on the order of 0.1 - 0.4 Newtons). Our objective is to increase the force capability of these actuators tenfold and use them to actively control a flexible structure.

Comparisons and Case Studies in Nonlinear Control

Over the past year, Doyle and coworkers have begun to formulate a unified approach to nonlinear stabilization. Initially, several design tools were considered and compared:

- control Lyapunov functions and inverse optimality
- "frozen" Riccati equations and nonlinear matrix inequalities
- gain scheduling using Quasi-LPV methods and LMIs
- Nonlinear model predictive control (MPC), in several variants
- differential flatness.

An important new development is a unified theoretical framework in which each method can be viewed as a special case, facilitating comparisons and extensions. Examples and counterexamples are generated using a "converse Hamilton-Jacobi-Bellman" tool. Several other methods serve as generic "straw men" for comparison, including nonlinear "H-infinity" optimal control, Jacobian linearization plus linear optimal control, and feedback linearization.

Generalizations of H_∞ Optimization

The H_∞ design methodology often leads to conservative designs because 1) plant uncertainty can only be accounted for in an approximate manner and 2) the fact that worst case design allows the disturbances to be unrealistic signals, such as pure sinusoids, which are poor models for many types of physical disturbances (such as sensor or thermal noise, wind gusts, for impulsive forces). In his thesis work, D'Andrea has extended the traditional H_∞ optimization problem to allow for general closed loop design objectives that address these two limitations. In particular, non-conservative, computational tractable, linear matrix inequality based methods for control design are developed for a certain class of physically motivated uncertain systems. In addition, these new techniques can accommodate constraints on the allowable disturbances, excluding unrealistic disturbances from the design process.

3 Personnel Supported

Faculty	Gary Balas, UMN John Doyle, Caltech Jerrold Marsden, Caltech Richard Murray, Caltech Stephen Wiggins, Caltech
Postdoctoral fellows	Brianno Coller, Caltech Ian Fialho, UMN Wang Sang Koon, Caltech
Graduate students	Anthony Blaom, Caltech Francesco Bullo, Caltech Raff D'Andrea, Caltech Alex Fax, Caltech Xiaolin Feng, Caltech Sonja Glavaski, Caltech Yun Huang, Caltech Michael Kantner, Caltech Sven Khatri, Caltech Mark Milam, Caltech Xiaoyun Zhu, Caltech
Others	David Chichka, Caltech (staff researcher)

4 Publications

Journal Publications

Appeared

[J1] A. M. Bloch and J. E. Marsden and G. Sánchez. Stabilization of relative equilibria of mechanical systems with symmetry. In M. Alber, B. Hu, and J. Rosenthal, editors, *Current and Future Directions in Applied Mathematics*, pages 43–64. Birkhäuser, 1997.

[J2] W. S. Koon and J. E. Marsden. Optimal control for holonomic and nonholonomic mechanical systems with symmetry and lagrangian reduction. *SIAM Journal of Control and Optimization*, 35:901–929, 1997.

[J3] W. S. Koon and J. E. Marsden. The Hamiltonian and Lagrangian approaches to the dynamics of nonholonomic systems. *Reports on Mathematical Physics*, 1997.

[J4] N. E. Leonard and J. E. Marsden. Stability and drift of underwater vehicle dynamics: Mechanical systems with rigid motion symmetry. *Physica D*, 105:103–162, 1997.

[J5] J. E. Tierno, R. M. Murray, J. C. Doyle, and I. M. Gregory. Numerically efficient robustness analysis of trajectory tracking for nonlinear systems. *Journal of Guidance, Control, and Dynamics*, 20(4):640–647, 1997.

Accepted

[J6] G. J. Balas. Synthesis of controllers for the active mass driver system in in the presence of uncertainty. *Earthquake Engineering and Structural Dynamics*, 1997. To appear.

[J7] R. D'Andrea. H-infinity optimal interconnections. *Systems and Control Letters*, 1998. To appear.

[J8] W. Lu, G. J. Balas, and E. B. Lee. Hankel norms, system integral invariance and comparison to standard induced norms. *IEEE Transactions on Automatic Control*, 1998. To appear.

[J9] M. van Nieuwstadt, M. Rathinam, and R. M. Murray. Differential flatness and absolute equivalence. *SIAM Journal of Control and Optimization*, 1997. To appear.

[J10] M. J. van Nieuwstadt. Rapid hover to forward flight transitions for a thrust vectored aircraft. *Journal of Guidance, Control, and Dynamics*, 1997. To appear.

[J11] M. J. van Nieuwstadt and R. M. Murray. Real time trajectory generation for differentially flat systems. *International Journal of Robust and Nonlinear Control*, 1997. To appear.

Submitted

[J12] G. J. Balas and P. M. Young. Sensor selection via closed-loop control objectives. *IEEE Transitions on Control Systems Technology*, 1997. Submitted.

[J13] I. Fialho, G. J. Balas, A. K. Packard, J. Renfrow, and C. Mullaney. Gain-scheduled lateral control of the f-14 aircraft during powered approach landing: A linear fractional approach. *Journal of Guidance, Control, and Dynamics*, 1997. Submitted.

Conference Proceedings

[C1] G. J. Balas. A flexible structure controls experiment. In *Proc. American Control Conference*, 1997.

[C2] G. J. Balas, I. Fialho, A. K. Packard, J. Renfrow, and C. Mullaney. On the design of lpv controllers for the f-14 lateral-directional axis during powered approach. In *Proc. American Control Conference*, 1997.

[C3] R. D'Andrea. Generalized l-2 synthesis: A new framework for control design. In *Proc. IEEE Control and Decision Conference*, pages 4655–4660, 1996.

[C4] R. D'Andrea. Necessary and sufficient conditions for gain scheduling subject to full-structured uncertainty. In *Proc. IEEE Control and Decision Conference*, pages 3760–3765, 1996.

[C5] J. C. Doyle, F. Paganini, R. D'Andrea, and S. Khatri. Approximate behaviors. In *Proc. IEEE Control and Decision Conference*, pages 688–693, 1996.

[C6] J. C. Doyle, J. Primbs, B. Shapiro, and V. Nevistić. Nonlinear games: examples and counterexamples. In *Proc. IEEE Control and Decision Conference*, pages 3915–3920, 1996.

[C7] I. Fialho, G. J. Balas, A. K. Packard, J. Renfrow, and C. Mullaney. F-14 lateral-directional axis during powered approach. In *Proc. American Control Conference*, 1997.

[C8] R. Freeman and J. Primbs. Control Lyapunov functions: New ideas from an old source. In *Proc. IEEE Control and Decision Conference*, pages 3926–3931, 1996.

[C9] Yun Huang and Wei Min Lu. Nonlinear optimal control: Alternatives to Hamilton-Jacobi equation. In *Proc. IEEE Control and Decision Conference*, 1996.

[C10] M. Kantner and J. A. Primbs. Nonlinear mpc lower bounds via robust simulation. In *Proc. American Control Conference*, 1997.

[C11] S. Khatri, R. D'Andrea, and J. C. Doyle. Uncertain hierarchical modeling. In *Proc. IEEE Control and Decision Conference*, pages 412–417, 1996.

[C12] R. Lind, V. Nalbantoglu, and G. J. Balas. Choosing sensor configuration for a flexible structure using full control synthesis. In *AIAA Conference on Guidance, Navigation, and Control*, 1997.

[C13] V. Nalbantoglu, J. Bokor, and G. J. Balas. System identification with orthonormal basis functions: An application to flexible structures. In *2nd IFAC Symposium on Robust Control Design*, Budapest, 1997.

[C14] V. Nevistić and J. A. Primbs. Model predictive control: breaking through constraints. In *Proc. IEEE Control and Decision Conference*, pages 3932–3937, 1996.

[C15] V. Nevistić and J. A. Primbs. Receding horizon quadratic optimal control: performance bounds for a finite horizon strategy. In *Proc. European Control Conference*, 1997.

[C16] A. Packard and M. Kantner. Gain Scheduling the LPV Way. In *Proc. IEEE Control and Decision Conference*, pages 3938–3941, 1996.

[C17] J. A. Primbs and V. Nevistić. MPC extensions to feedback linearizable systems. In *Proc. American Control Conference*, 1997.

Theses

[T1] R. D'Andrea. *Generalizations of H-Infinity Optimization / Control of Rotating Stall.* PhD thesis, California Institute of Technology, 1996.

5 Interactions and Transitions

Meetings and conferences

All personnel have attended a variety of conferences including CDC, ACC, and GNC. In addition, the following specific activities were performed:

- Murray attended a planning workshop on UAVs at Wright-Patterson AFB in August 1996.
- Murray attended a workshop on the Airborne Laser at Kirtland AFB in October 1996.
- Doyle organized an invited session at the 1997 Conference on Decision and Conrol presenting case studies that highlighted the application of variety of nonlinear control design methodologies to optimal control problems.

- Doyle organized a workshop at the 1997 American Control Conference title "Nonlinear Control: Comparisons and Case Studies". Over 40 people attended.
- Balas co-organized a workshop on linear parameter varying control at the 1997 American Control Conference (see below).

Consulting and advisory functions

- Murray is consulting for United Technologies Research Center in the area of nonlinear control of combustion instabilities.
- Marsden is involved in ongoing discussions with Martin Lo (and others) at JPL on the use of "dynamical channels" and optimization/control methods for trajectory generation. They make use of the general PRET research program on control of mechanical systems. We believe that these have benefited JPL in positive ways and contribute to new trajectory plans (e.g., with the Genesis mission).

Transitions

1. **Performer:** Prof Gary Balas, University of Minnesota, (612) 625-6857

   Customer: Controls community

   Contact: Dr. Mike Masten, (972) 995-7986

   Results: A short course at the 1997 American Controls Conference on parameter-dependent control and its application to real world control problems (Co-Organizer with Prof. Andy Packard). This course provided attendees with a brief background on robust control, and linear matrix inequalities (LMIs). The underlining theory for LPV systems was presented as well as the solution to several gain-scheduled control problems that arise in the LPV framework. To provide hands-on experience analyzing and synthesizing controllers using linear parameter varying (LPV) techniques, we provided one PC computer for every 3 attendees. We have found this helped reinforce the theory and results presented in the lectures. A total of 55 people from a variety of areas attended this course. This was nearly twice the number of attendees than the other short courses.

   Application: Application of LPV methods to the F-14 lateral-directional axis powered-approach flight control system, design of missile autopilots, a chemical process and turbofan engines were presented.

2. **Performer:** Prof Gary Balas, University of Minnesota, (612) 625-6857

   Customer: MUSYN Inc. and the Naval Air Warfare Center, Patuxent River, MD.

   Contact: Chris Mullaney, (301) 342-7720

   Results: Application of LPV techniques to the design of power-approach lateral-directional flight control law for the F-14. These control designs were compared with the newly developed digital flight control system (DFCS) which will be introduced into the fleet in the coming years. In piloted simulations of the power-approach landing, lead test pilot Lt. Scott Kelly and Lt. Poindexter of the DFCS flights, gave the LPV gain-scheduled designs a Cooper-Harper rating of 4. We have redesign these LPV controllers and are waiting for an opportunity to test them in pilot-in-the-loop simulation.

Application: The high performance, robust, gain-scheduled multivariable controllers perform as well as the current designs. These results lend support to these techniques to design current and future flight control systems. As systems become more highly coupled, robust, gain-scheduled multivariable techniques offer a major benefit over classical methods in terms of achievable performance, robustness and overall safety. LPV software will soon be available in commercial software (specifically the mu-Analysis and Synthesis Toolbox) provide industry and government labs usable advance control analysis and design tools.

3. **Performer:** Richard M. Murray, California Institute of Technology, (626) 395-6460

   Customer: Boeing North American

   Contact: Daniel Hill, (714) 762-1151

   Results: A collaborative project has been established between Boeing North American (formerly Rockwell) and Caltech to apply modern robust control techniques to stabilization of an electrostatic gyro and to evaluate performance enhancements due to use of modern, multi-variable control techniques. Boeing has committed internal funds for FY97 and FY98 to provide technical support and experimental facilities. Initial meetings have been conducted to establish the overall direction of the collaboration and a student has been identified to work on the project. This work is jointly supported by the AFOSR PRET center the AFOSR MURI center on Robust Virtual Engineering.

   Application: High performance control of an electrostatic gyro, an inertial guidance sensor marketed by Boeing and currently used for ship and submarine applications.

6 Honors and Awards

Jerrold Marsden

June 1997	Election to the American Academy of Arts and Science

Richard Murray

May 1997	Plenary lecture, SIAM Conference on Applications of Dynamical Systems
June 1997	Donald P. Eckman Award

New Discoveries, Inventions, or Patent Disclosures

None.