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# Robust Nonlinear Control Theory with Applications to Aerospace Vehicles

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Progress Report
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# 1 Status of Effort

This was the first full year of this new program. We continued our work in robust linear control theory, with increased emphasis on analysis of implicit systems and unified modeling, system identification, analysis and design. A high point of this research was the dissertation of Paganini, which solved the open problem of robust $\mathcal{H}_2$ synthesis and was awarded the Clauser prize for the best thesis at Caltech for 1995–96. Continued work in robustness analysis for nonlinear systems has also shown excellent promise and research is continuing in this area.

Several new results were established for control of mechanical systems in the areas of controllability, stability, and trajectory tracking, laying a solid foundation for future work. Initial work on using techniques from dynamical systems theory has concentrated on the role of normal forms in establishing global strategies for control in the presence of bifurcations.

We have applied the results from our research to a variety of systems. Caltech continues to make active use of its flight control experimental facility to validate and motivate new theory. UMN has applied control techniques from this program to both university experiments and industrial problems, most notably the control of an F14 in piloted simulation.

# 2 Accomplishments

## 2.1 Integrated modeling, identification, analysis, and design.

The use of mathematical models to design large systems involves various instances and possibly iterations of modeling, identification, system design and controller synthesis, and various forms of analysis and simulation. We are developing a variety of techniques for attacking individual problems as well as integration of these problems.

- New techniques have been developed for modeling of white noise by employing standard statistical tests in order to identify a typical set, and performing subsequent analysis in a worst-case setting [T2]. The combination of white noise and unmodeled dynamics allows a solution to the the robust $\mathcal{H}_2$ performance problem, which is rooted in the origins of robust control theory.

- The $\mathcal{H}_\infty$ framework was extended in [C3] and [C4] to allow for general performance constraints when designing controllers in an $l_2$ setting. Some of these extensions include synthesizing controllers for systems subject to a mix of arbitrary $l_2$ disturbances and deterministic noise disturbances [C5], and the synthesis of controllers for plants subject to full structured uncertainty [C3]. These results can in turn be combined with other linear matrix inequality based solutions, such as the linear parameter varying framework [C6].

- A numerically efficient algorithm has been developed for providing a lower bound on robust performance of a nonlinear system along a prespecified trajectory [J11]. This algorithm has been applied to an F16 model and indicates high potential for robustness analysis of this type.

- Developed a technique to compute upper bounds on robust performance by approximating a nonlinear system by a rational system which can be represented using linear fractional transformations [C19]. Initial testing indicates that this approach has poor growth characteristics and hence is computationally difficult.

- We have developed robust simulation theory and code for discrete time nonlinear systems. Robust simulation allows one to simulate all possible trajectories for a nonlinear system. By simulating all trajectories, one can obtain upper bounds on worst case nonlinear performance [C10].

### Nonlinear Control of Mechanical Systems

A feature of the many aerospace systems is their mechanical nature. Driven by new theoretical results in Lagrangian control systems, a more detailed understanding of the role of symmetries, constraints, and external forces is emerging which has important implications for many of the specific systems considered in this program. By properly accounting for the second order nature of mechanical systems, it is possible to analyze the control of these systems in a way which remains true to the underlying geometry and allows the nonlinear nature of the mechanical system to be exploited to a fuller extent than previously possible.

- New tools have been developed for characterizing and classifying controllability of mechanical systems, concentrating on the case of *configuration controllability* [J3, J8]. This work provides a geometric basis for future work in nonlinear control of mechanical systems.
- We have extended the stability theory of relative equilibria for mechanical systems with symmetry, concentrating on systems that have a noncompact symmetry group, such as the group of Euclidean motions, and with relative equilibria for such symmetry groups [J7]. For these systems with rigid motion symmetry, one gets stability but possibly with drift in certain rotational as well as translational directions. The setting of this work should prove useful for the control of underwater vehicles and related mechanical systems.

### Normal Forms for Nonlinear Control Systems

A key paradigm in dynamical systems theory is the use of nonlinear normal forms. Various low order models are available that capture the types of global nonlinear behavior and bifurcation behavior that exist in physical systems. We are applying existing techniques to control problems as well as developing new machinery for understanding the role of normal forms and normal systems in nonlinear control.

- Developed algorithms for real-time trajectory generation for differentially flat systems [C20, C22, T1]. These algorithms allow generation of feasible state space trajectories and are useful in a variety of contexts where aggressive motion of the system is required.
- In conjunction with the AFOSR-PRET program on aeroengines, we have applied tools in bifurcation theory to study the nonlinear dynamics of a simple model of an axial flow compressor in the presence of rotating stall and surge instabilities. This analysis is a first step in using the non-local dynamics of the system to provide controllers which exploit the natural response of the system to achieve performance objectives.
- We have begun to develop some low order models for flutter (in turbomachines) with the goal of developing a deeper theoretical understanding of mistuning effects in this type of system.

## Applications of Robust Control

- Implemented a variety of improvements for the Caltech ducted fan, a small nonlinear flight control experiment, including the addition of a wing and redesign of the stand to give performance much closer to that of typical aircraft systems.

- A software library for real-time trajectory generation has been developed and applied to the Caltech ducted fan [C21, J10, T1]. This library can be directly applied to systems which are differentially flat and seems to work well on systems which are only approximately differentially flat (such as the ducted fan).

- Controller synthesis algorithms have been developed for the optimal constant scaling $\mathcal{H}_\infty$ full information problem with real and complex time-invariant and time-varying uncertainty [J1]. These results provide an optimal lower bound on the performance robustness of $\mu$ synthesis output feedback controller that are designed. Controllers for a missile autopilot and the UMN structure have been designed and compared with output feedback controllers. $\mu$ analysis and synthesis methods were also successfully applied to the design of powered-approach lateral-directional flight controllers for the F-14 and a 3M extrusion process. (More comments about these applications in the technology transfer section.)

- A nonlinear mathematical model of the stress-strain behavior of shape memory alloy wires that includes the behavior of the material within the pseudoelastic hysteresis loop was derived [C18]. A frequency domain model is also posed for the shape memory elements. This model is useful for the analysis and design of active vibration attenuation systems. Experimental results demonstrate the effectiveness of shape memory wires in augmenting passive damping of large flexible structures with as much as a factor of 6 increase in the equivalent viscous damping level of vibrational modes. Robust multivariable control techniques, specifically $\mu$-synthesis, were used to synthesize controllers for the UMN structure experiment augmented with shape memory alloy (SMA) wires for passive damping [C17]. The SMA wires introduce extra damping in the lower modes of the structure. This results in increased performance and improved robustness of the multivariable controllers designed with the SMA elements.

# 3 Personnel Supported

| | |
|---|---|
| Faculty | Gary Balas, UMN<br>John Doyle, Caltech<br>Jerrold Marsden, Caltech<br>Richard Murray, Caltech<br>Stephen Wiggins, Caltech |
| Postdoctoral fellows | Brianno Coller, Caltech<br>Geir Dellarud, Caltech<br>Ian Fialho, UMN<br>Willem Sluis, Caltech |
| Graduate students | Carolyn Beck, Caltech<br>Anthony Blaom, Caltech<br>Raff D'Andrea, Caltech<br>Xiaolin Feng, Caltech<br>Sonja Glavaski, Caltech<br>Yun Huang, Caltech<br>Michael Kantner, Caltech<br>Sven Khatri, Caltech<br>Rick Lind, UMN<br>Mark Milam, Caltech<br>John Morris, Caltech<br>Matthew Newlin, Caltech<br>Michiel van Nieuwstadt, Caltech<br>Fernando Paganini, Caltech<br>Peter Thomson, UMN<br>Xiaoyun Zhu, Caltech |

# 4 Publications

## Journal Publications

### Appeared

[J1] G. J. Balas, R. Lind, and A. K. Packard. Optimally scaled $\mathcal{H}_\infty$ full information control with real uncertainty: Theory and application. *Journal of Guidance, Control, and Dynamics*, 19(4):854–862, 1996.

### Accepted

[J2] W. S. Koon and J. E. Marsden. Optimal control for holonomic and nonholonomic mechanical systems with symmetry and Lagrangian reduction. *SIAM Journal of Control and Optimization*, 1996. To appear.

[J3] A. D. Lewis and R. M. Murray. Configuration controllability of simple mechanical control systems. CDS Technical Report CIT/CDS 95-015, Caltech Control and Dynamical Systems, 1995. To appear, *SIAM J. Control and Optimization*.

[J4] W. Lu and J. C. Doyle. Robustness analysis and synthesis of nonlinear uncertain systems. *IEEE Transactions on Automatic Control*, 1996. To appear.

[J5] W. Sluis, A. Banaszuk, J. Hauser, and R. M. Murray. A homotopy algorithm for approximating geometric distributions by integrable systems. *Systems and Control Letters*, 1996. To appear.

[J6] P. Thomson, G. J. Balas, and P. H. Leo. The effects of shape memory alloy pseudoelastic hysteresis models on structural systems. *J. Intelligent Material Systems and Structures*, 1996. To appear.


**Submitted**


[J7] N. E. Leonard and J. E. Marsden. Stability and drift of underwater vehicle dynamics: Mechanical systems with rigid motion symmetry. Technical report, Princeton Mechanical Engineering, 1995. Submitted to *Physica D*.

[J8] A. Lewis and R. M. Murray. Decompositions for control systems on manifolds with an affine connection. *Systems and Control Letters*, 1996. Submitted.

[J9] W. Lu and J. C. Doyle. Disturbance rejection of persistent bounded disturbances. *IEEE Transactions on Automatic Control*, 1996. Submitted.

[J10] M. J. van Nieuwstadt and R. M. Murray. Real time trajectory generation for differentially flat systems. *International Journal of Robust and Nonlinear Control*, 1996. Submitted.

[J11] J. E. Tierno, R. M. Murray, J. C. Doyle, and I. M. Gregory. Numerically efficient robustness analysis of trajectory tracking for nonlinear systems. CDS Technical Report CIT/CDS 95-032, Caltech, 1995. (submitted, *J. Guidance, Control, and Dynamics*).


## Conference Proceedings


[C1] G. J. Balas, A. K. Packard, R. T. M'Closkey, J. Renfrow, and C. Mullaney. Design of controllers for the F-14 aircraft lateral-directional axis during powered approach. In *AIAA Conference on Guidance, Navigation, and Control*, 1996.

[C2] A. Banaszuk, W. Sluis, J. Hauser, and R. M. Murray. On measures of non-integrability of Pfaffian systems. In *Proc. IFAC World Congress*, 1996.

[C3] R. D'Andrea. H-infinity optimization with spatial constraints. In *Proc. IEEE Control and Decision Conference*, pages 4327–4332, 1995.

[C4] R. D'Andrea. Generalized l-2 synthesis: A new framework for control design. In *Proc. IEEE Control and Decision Conference*, 1996. To appear.

[C5] R. D'Andrea. LMI approach to mixed H-2 and H-infinity performance objective controller design. In *Proc. IFAC World Congress*, 1996.

[C6] R. D'Andrea. Necessary and sufficient conditions for gain scheduling subject to full-structured uncertainty. In *Proc. IEEE Control and Decision Conference*, 1996. To appear.

[C7] R. D'Andrea and J. C. Doyle. Full information and full control in a behavioral context. In *Proc. IEEE Control and Decision Conference*, 1996. To appear.

[C8] J. C. Doyle and R. M. Murray. Robust nonlinear control theory with applications to aerospace vehicles. In *Proc. IFAC World Congress*, 1996.

[C9] Y. Huang and W. Lu. Hamilton-Jacobi equations and nlmis. In *Proc. IEEE Control and Decision Conference*, 1996.

[C10] M. Kantner and J. C. Doyle. Robust simulation and nonlinear performance. In *Proc. IEEE Control and Decision Conference*, 1996. To appear.

[C11] S. D. Kelly and R. M. Murray. The geometry and control of dissipative systems. In *Proc. IEEE Control and Decision Conference*, 1996. Submitted.

[C12] A. D. Lewis and R. M. Murray. Equilibrium controllability for a class of mechanical systems. In *Proc. IEEE Control and Decision Conference*, 1995.

[C13] R. Lind and G. J. Balas. Optimal full information synthesis for flexible structures implemented on cray supercomputers. In *AIAA Conference on Guidance, Navigation, and Control*, 1996.

[C14] R. M. Murray. Trajectory generation for a towed cable flight control system. In *Proc. IFAC World Congress*, 1996.

[C15] R. M. Murray, M. Rathinam, and W. Sluis. Differential flatness of mechanical control systems: A catalog of prototype systems. In *ASME International Mechanical Engineering Congress and Exposition*, San Francisco, November 1995.

[C16] V. Nalbantoğlu, G. J. Balas, and P. Thomson. The role of performance criteria selection in the control of flexible structures. In *AIAA Conference on Guidance, Navigation, and Control*, 1996.

[C17] P. Thomson, V. Nalbantoğlu, and G. J. Balas. Shape memory alloys for active control of flexible structures. In *First European Conference on Structural Control*, 1996.

[C18] P. Thomson, V. Nalbantoğlu, and G. J. Balas. Shape memory alloys for augmenting damping of flexible structures. In *AIAA Conference on Guidance, Navigation, and Control*, 1996.

[C19] J. E. Tierno and R. M. Murray. Robust performance analysis for a class of uncertain nonlinear systems. In *Proc. IEEE Control and Decision Conference*, 1995.

[C20] M. van Nieuwstadt and R. M. Murray. Approximate trajectory generation for differentially flat systems with zero dynamics. In *Proc. IEEE Control and Decision Conference*, pages 4224–4230, 1995.

[C21] M. van Nieuwstadt and R. M. Murray. Fast mode switching for a thrust vectored aircraft. In *Multiconference on Computational Engineering in Systems Applications*, Lille, France, July 1996.

[C22] M. van Nieuwstadt and R. M. Murray. Real time trajectory generation for differentially flat systems. In *Proc. IFAC World Congress*, 1996.

### Theses

[T1] M. van Nieuwstadt. *Trajectory Generation for Nonlinear Control Systems.* PhD thesis, California Institute of Technology, 1996.

[T2] F. Paganini *Sets and Constraints in the Analysis of Uncertain Systems.* PhD thesis, California Institute of Technology, 1996.

[T3] P. Thomson. *Shape Memory Alloys for Structural Control.* PhD thesis, University of Minnesota, 1996.

## 5 Interactions and Transitions

### Meetings and conferences

All personnel have attended a variety of conferences including CDC, IFAC, GNC, and MTNS. In addition, the following specific activities were performed:

- Doyle co-organized a two day short course on implicit systems at the 1995 CDC in New Orleans.
- Murray attended the final review of the Multivariable Control Guidelines at Wright Patterson AFB in May 1996.
- Doyle and Murray organized a poster session at IFAC in July 1996 highlighting new research results funded by this program.
- Caltech held a PRET annual review/workshop in March 1996, in conjunction with the AFOSR contractors meeting. Representatives from nine aerospace companies and a variety of government and academic researchers attended the workshop.

### Consulting and advisory functions

- Coller, Marsden, and Murray are interacting with UTRC on possible applications of Hopf-Hopf analysis and flutter models.
- Murray visited Honeywell, McDonnell-Douglas, Northrop and Hughes in Fall 1996 to discuss PRET-related activities. This has led to interactions with Kevin Wise at McDonnell Douglas as part of Air Force sponsored activities in control of tailless aircraft.
- Jorge Tierno received his PhD at Caltech in September 1995 and accepted a position at the Honeywell Technology Center. Part of Tierno's work at Honeywell involves applying and extending the results of his thesis on robustness analysis for nonlinear systems.

### Transitions

1. **Performer:** Prof Gary Balas, University of Minnesota, (612) 625-6857

   **Customer:** 3M

   **Contact:** Dr. Steve Mohn (612) 733-4639

   **Results:** Application of robust multivariable control analysis and design techniques to the control of extrusion processes.

**Application:** Control design approach useful for the extrusion of thin films, composite materials and other advanced material that requires numerous, actuators and sensors.

2. **Performer:** Prof Gary Balas, University of Minnesota, (612) 625-6857

   **Customer:** MUSYN Inc. and the Naval Air Warfare Center, Patuxent River, MD.

   **Contact:** Chris Mullaney, (301) 342-7720

   **Results:** Application of structured singular value synthesis technique via D-K iteration using the mu-Analysis and Synthesis Matlab Toolbox to the design of power-approach lateral-directional flight control law for the F-14. These control designs were compared with the newly developed digital flight control system (DFCS) which will be introduced into the fleet in the coming years. In piloted simulations of the power-approach landing, lead test pilot Lt. Scott Kelly of the DFCS flights, commented about the $\mu$ control design that "This is how I thought the airplane should fly. I thought it was the best flight control law I've flown in the F-14. The AFCS has a lot of deficiencies, the DFCS is a big improvement over that, but in the limited look I had in the simulator I really like what I saw of this design."

   **Application:** The high performance, robust multivariable controllers significantly out performed the current designs. These results lend support to the use of advanced multivariable control design techniques to design of current and future flight control systems. As systems become more highly coupled, robust multivariable techniques offer a major benefit over classical methods in terms of achievable performance, robustness and overall safety. The availability of commercial software (specifically the mu-Analysis and Synthesis Toolbox) provide industry and government labs usable advance control analysis and design tools.

## 6 Honors and Awards

*Jerrold Marsden*

1996 Plenary (opening) lecture, Geometric Methods in Mathematical Physics, Goslar, Germany, July, 1996

*Richard Murray*

1996 ONR Young Investigator Award

## New Discoveries, Inventions, or Patent Disclosures

None.