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THE DYNAMIC SENSOR COVERAGE PROBLEM

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Abstract: We introduce a theoretical framework for the dynamic sensor coverage problem for a simple case with multiple discrete time linear dynamical systems located in different spacial locations. The objective is to keep an appreciable estimate of the states of the systems at all times by deploying a few mobile sensors. The sensors are assumed to have a limited range and they implement a Kalman filter to estimate the states of all the systems. The motion of the sensor is modeled as a discrete time discrete state Markov chain. Based on some recent results on the Kalman filtering problem with intermittent observations by Sinopoli et. al., we derive conditions under which a single sensor fails to solve the coverage problem. We also give conditions under which we can guarantee that a single sensor is enough to solve the dynamic coverage problem

Keywords: Dynamic Sensor Coverage, Markov Chain, Kalman Filter

1. INTRODUCTION

Sensor coverage is the problem of deploying multiple sensors in an unknown environment for the purpose of automatic surveillance, cooperative exploration and target detection. Recent years have witnessed increased interest among the communication, control and robotics researchers in the area of mobile sensor networks. Each individual node in such a network has sensing, computation, communication and locomotion capabilities. When the environment is rapidly changing finding an efficient deployment strategy becomes a key issue for any application.

Coverage can be static (fixed sensors) or dynamic (mobile sensors). Static sensor coverage is desirable if the area to be covered is less than the union of the ranges of the sensor nodes. Static sensor coverage problem has been considered in (Cortes *et al.*, 2004*b*), (Cortes *et al.*, 2004*a*) and in the references there in. The dynamic sensor coverage becomes necessary when a limited number of sensors are available and the area of interest can not be covered by a static configuration of sensors.

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There have been attempts to empirically solve the dynamic coverage problem using simulations and actual robots (Batalin and Sukhatme, 2002) but a sound theoretical base is still missing in the literature.

In this paper we consider N discrete time linear systems located at different points in space. One may think of dividing the area under consideration using a grid and then these N systems can be thought to represent the dynamics of local environment change at the grid points. To begin with, we consider the case when a single sensor is employed. The sensor maintains discrete time Kalman filter estimates of the states of all the N systems. In order to model the limited range of the sensor, we constrain the sensor to receive measurements only for the system where it is physically located at that time instant. All the tools developed in this paper can be applied to the case where multiple grid points fall in the sensory range and hence the sensor receives measurements from more than one system, with little modifications and is left as a future research direction. For a system where the sensor is located, the sensor implements both the time update and measurement update laws of the Kalman filter. For all the other systems for which the sensor did not receive any measurements, only the time update law is implemented at that time instant. The motion of the sensor is governed by a discrete time discrete state (DTDS) Markov chain, as shown in fig 1. For successful coverage the sensor needs to hop from one system to another such that the error covariance matrices of the estimates of states of all the N systems are bounded at all times. Intuition tells us that the sensor should spend more time at a location where the environment is changing rapidly than the one where the dynamics are relatively slow.

If multiple locations have fast evolving systems, one sensor may not be enough to solve the coverage problem. We provide conditions under which such a scenario results. Under a different set of conditions we prove that a single sensor is enough to solve the sensor coverage problem.

2. PROBLEM DESCRIPTION

Consider N independently evolving linear discrete time systems, whose dynamics are given by

$$\begin{cases} x_{i,t+1} = A_i x_{i,t} + w_{i,t} \\ y_{i,t} = C_i x_{i,t} + v_{i,t} \end{cases} \quad (1)$$

where $x_{i,t},\ x_{i,t+1},\ w_{i,t} \in \mathbb{R}^{n_i}$ and $y_{i,t},\ v_{i,t} \in \mathbb{R}^{m_i}$, w_i and v_i are Gaussian random vectors with zero mean and covariance matrices Q_i and

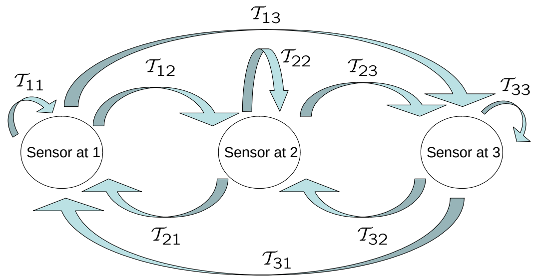

Fig. 1. Markov state diagram of sensor motion

R_i respectively and i takes values in the set $\{1, 2, 3, \cdots, N\}$.

As already mentioned, the space to be covered can be discretized using a grid and the above N systems can be thought to represent the dynamics of certain local variables at the grid points. These variables can be temperature, barometric pressure in case of weather monitoring, threat emergence rate in case of surveillance and congestion measure in the case of a network. Cite some references here.

In reality the independent evolution of the systems assumption may not always hold, as the dynamics of systems proximate in space may be highly dependent or even coupled. We are currently working on the coupled environment case.

We assume that the motion of the sensor is governed by a DTDS Markov chain. The number of states is N. If the sensor is in state i at time t it only has access to the measurement of the ith system at that time. The state transitions occur at a fixed time interval which is assumed to be the same as the sampling period of the N systems without any loss of generality. An example discrete time discrete state Markov chain for $N = 3$ is shown in figure 1. We will refer to the transition probability matrix as $\mathcal{T}$. $\mathcal{T}_{ij}$ is the probability that the sensor will be at location j at time $t+1$ given that it is in location i at time t.The matrix $\mathcal{T}$ and the relation between the probability mass functions at time t and $t+1$, for the state transition diagram in Fig. 1 is shown in Eq. (2). There may be physical constraints on the motion of the sensor as for example the sensor can not move between two systems located far away in space in one time interval. Such restrictions can be imposed by making the corresponding transition probability between such states to be equal to zero.

Let $\pi_i(t)$ be the probability of the sensor being in the ith location at time t. Then we have eqn. (2).

Markov chains have been used earlier for search and surveillance problems in the operations research community (Jeffcoat, 2004), (Stone, 1989).

The sensor runs N Kalman filters one for each of the N systems. For system i the time update equations of the Kalman filter are implemented at all time instants, whereas the measurement update equations are implemented only at those time instants when the sensor happens to be at location i.

$$\pi(t+1) = \begin{bmatrix} \pi_1(t+1) \\ \pi_2(t+1) \\ \pi_3(t+1) \end{bmatrix}^T = \begin{bmatrix} \pi_1(t) \\ \pi_2(t) \\ \pi_3(t) \end{bmatrix}^T \underbrace{\begin{bmatrix} \mathcal{T}_{11} & \mathcal{T}_{12} & \mathcal{T}_{13} \\ \mathcal{T}_{21} & \mathcal{T}_{22} & \mathcal{T}_{23} \\ \mathcal{T}_{31} & \mathcal{T}_{32} & \mathcal{T}_{33} \end{bmatrix}}_{\mathcal{T}} \quad (2)$$

Let $I_{i,t}$ be the indicator function describing whether or not the sensor is at location i.

$$\mathbb{P}[I_{i,t} = 1] = \pi_i(t) \quad (3)$$

We model the variance of the measurement noise for the ith system in the following manner

$$Var(v_{i,t}) = \begin{cases} R_i, & I_{i,t} = 1 \\ {\sigma_i}^2 I, & I_{i,t} = 0 \end{cases}.$$

When the sensor is not at the location i no observation is made for system i and this corresponds to the limiting case of $\sigma \to \infty$. Following a similar approach as in (Sinupoli *et al.*, 2003) we get the following Kalman filter equations:

$$\hat{x}^-_{i,t+1} = A_i \hat{x}_{i,t} \quad (4)$$

$$P^-_{i,t+1} = A_i P_{i,t} A'_i + Q_i \quad (5)$$

$$\hat{x}_{i,t+1} = \hat{x}^-_{i,t+1} + I_{i,t+1} P^-_{i,t+1} C'_i \times (C_i P^-_{i,t+1} C'_i + R_i)^{-1} (y_{i,t+1} - C_i \hat{x}^-_{i,t+1}) \quad (6)$$

$$P_{i,t+1} = P^-_{i,t+1} - I_{i,t+1} P^-_{i,t+1} C'_i \times (C_i P^-_{i,t+1} C'_i + R_i)^{-1} C_i P^-_{i,t+1} \quad (7)$$

Eqns. (4) and (5) are the time update relations for the estimate and the error covariance. It can be clearly seen from eqns. (6) and (7) that the measurement update is performed only when the sensor is at location i.

Using the above equations the recursive relation for the a priori error covariance matrix can be written as

$$P^-_{i,t+1} = A_i P^-_{i,t} A'_i + Q_i - I_{i,t+1} A_i P^-_{i,t} C'_i (C_i P^-_{i,t} C'_i + R_i)^{-1} C_i P^-_{i,t} A'_i \quad (8)$$

For the rest of this section we will drop the $-$ superscript from $P^-_{i,t}$.

An important observation is that eq.(8) is stochastic in nature due to presence of the random variable $I_{i,t+1}$. We now have N of these stochastic recursive equations, one for each of the N systems. So to maintain an appreciable estimate of the states of all N systems we would want that $\lim_{t\to\infty} \mathbb{E}[P_{i,t}]$ remains bounded for all i.

Since both $I_{i,t+1}$ and $P_{i,t}$ are random variables, we know that

$$\mathbb{E}[P_{i,t+1}] = \mathbb{E}\big[\mathbb{E}[P_{i,t+1} | P_{i,t}]\big] \quad (9)$$

where the inner expectation operator is over $I_{i,t+1}$ and the outer expectation is over $P_{i,t}$. Therefore

$$\begin{aligned} \mathbb{E}[P_{i,t+1}] &= \mathbb{E}[A_i P_{i,t} A'_i + Q_i - \pi_i(t+1) A_i P_{i,t} C'_i (C_i P_{i,t} C'_i + R_i)^{-1} C_i P_{i,t} A'_i] \\ &= \mathbb{E}[g_{\pi_i(t+1)}(P_{i,t})] \end{aligned} \quad (10)$$

where

$$g_{\pi_i(t+1)}(X) = A_i X A'_i + Q_i - \pi_i(t+1) A_i X C'_i (C_i X C'_i + R_i)^{-1} C_i X A'_i \quad (11)$$

The above is a modified algebraic Riccati equation (MARE). As in (Sinupoli *et al.*, 2003), for $0 < \overline{\pi}_i < 1$ we define another MARE as

$$g_{\overline{\pi}_i}(X) = A_i X A'_i + Q_i - \overline{\pi}_i A_i X C'_i (C_i X C'_i + R_i)^{-1} C_i X A'_i \quad (12)$$

Its easy to observe that the difference between the MAREs (11) and (12) is that for (11) the random process $I_{i,t+1}$ is non-stationary and hence $\pi_i(t+1)$ is time varying, and it varies according to a DTDS Markov chain as shown in eqns. (2) and (3). We will now prove that to analyze the properties of $\mathbb{E}[P_{i,t}]$ as $t \to \infty$, its sufficient to work with the steady state probability density of the Markov chain but before that we need some basic definitions and results from the DTDS Markov chain literature.

Definition 1. A DTDS Markov chain is called *irreducible* if, starting from any one of the states, it is possible to get to any other state (not necessarily in one jump) with a non-zero probability.

Definition 2. A state i of a Markov chain X has period d if, given that $X_0 = i$, we can only have $X_n = i$ when n is a multiple of d. We call i *periodic* if it has some period > 1. If the markov chain X is irreducible, then either all states are periodic or none are.

Definition 3. Suppose X is a DTDS Markov chain with finite number of states, X is ergodic if

- X is irreducible
- X is not perodic.

Lemma 4. (Ergodic Theorem). If a DTDS Markov chain is ergodic, with transition matrix $\mathcal{T}$, then there is exactly one probability vector $\overline{\pi}$ which satisfies

$$\overline{\pi}^T = \overline{\pi}^T \mathcal{T}$$

In addition, for each i and j,

$$\mathbb{P}(X_n = j | X_0 = i) \rightarrow \overline{\pi}_j.$$

We need the following properties of the function $g_\pi(X)$

Lemma 5. Monotonicity. If $0 \le X \le Y$, then $g_\pi(X) \le g_\pi(Y)$

PROOF. See (Sinupoli *et al.*, 2003)

Lemma 6. Concavity. If $\beta \in [0,1]$, then $g_\pi(\beta X + (1-\beta)Y) \ge \beta g_\pi(X) + (1-\beta) g_\pi(Y)$.

PROOF. See (Sinupoli *et al.*, 2003)

Lemma 7. If X is a random variable then

$$\mathbb{E}[g_\pi(X)] \le g_\pi(\mathbb{E}(X)).$$

PROOF. From lemma 6, we know that $g_\pi(X)$ is a concave function of X, therefore the result holds by Jensens's Inequality.

Lemmas 5, 6 and 7 all hold for π time varying and constant. So π in the above results can be both $\pi(t+1)$ and $\overline{\pi}$.

Lemma 8. Suppose $\pi(t)$ varies according to an ergodic Markov chain and $\lim_{t\to\infty} \pi_i(t) = \overline{\pi}_i$ if (12) remains bounded for all positive semi definite initial conditions, then (11) remains bounded for all positive semi definite initial conditions

PROOF. If $P_{i,0} = Y^0 = 0$ then $P_{i,1} = Y^1 = g_{\pi_i(1)}(Y^0) = Q_i \ge 0$. Hence $Y^1 \ge Y^0$, therefore from lemma 5 $g_{\pi_i(1)}(Y^1) \ge g_{\pi_i(1)}(Y^0)$ or $P_{i,2} = Y^2 \ge Y^1 = P_{i,1}$. Using an induction argument we have

$$0 = Y^0 \le Y^1 \le Y^2 \le Y^3 \le \cdots$$

Let for some $t_f \ge 0$, $\pi_i(t_f) = \overline{\pi}_i$, and we know that $P_{i,t_f} = Y^{t_f} \ge 0$. For $t \ge t_f$ both MAREs (11) and (12) have the same recursive relation, and since (12) converges for all positive semi definite initial conditions it converges for $P_{i,t_f} \ge 0$.

If $P_{i,0} = Z^0 \ge 0$, by lemma 5 we know that $g_{\pi_i(1)}(Z^0) \ge g_{\pi_i(1)}(Y^0)$, which implies $Z^1 \ge Y^1$ and thus $Z^{t_f} \ge Y^{t_f} \ge 0$. Again for $t > t_f$ both recursive relations are the same and from the assumption of the lemma we know that (12) converges for all positive semi definite initial conditions.

In order to find the conditions under which $\lim_{t\to\infty} \mathbb{E}[P_{i,t}]$ is unbounded, we define the following recursive relation

$$h_{\pi_i(t)}(X) = (1-\pi_i(t)) A_i X A_i' + Q_i \qquad (13)$$

Lemma 9. For $Y \ge X \ge 0$

(a)$h_{\pi_i(t)}(Y) \ge h_{\pi_i(t)}(X)$.
(b)$g_{\pi_i(t)}(X) \ge h_{\pi_i(t)}(X)$.
(c) If X is a random variable $\mathbb{E}[g_{\pi_i(t)}(X)] \ge h_{\pi_i(t)}(\mathbb{E}[X])$.

PROOF.

(a)$h_{\pi_i(t)}(Y) - h_{\pi_i(t)}(X) = (1-\pi_i(t))A_i(Y - X)A_i' \ge 0$.
(b) See (Sinupoli *et al.*, 2003).
(c) By linearity of the expectation operator.

Lemma 10. Suppose $\pi(t)$ varies according to an ergodic Markov chain and $\lim_{t\to\infty} \pi_i(t) = \overline{\pi}_i$. Let A_i be unstable and $(A_i, {Q_i}^{\frac{1}{2}})$ be controllable, then $\lim_{t\to\infty} \mathbb{E}[P_{i,t}]$ is unbounded for some $P_{i,0} \ge 0$ if

$$\overline{\pi}_i < 1 - \frac{1}{{\alpha_i}^2}$$

where $\alpha_i = \max_j |\lambda_{i,j}|$, and $\lambda_{i,j}$ are the eigenvalues of A_i.

PROOF. Lets consider the recursive equation

$$Y^{t+1} = h_{\pi_i(t+1)}(Y^t) = (1-\pi_i(t+1)) A_i Y^t A_i' + Q_i \qquad (14)$$

For $t > t_f$, $\pi_i(t) = \overline{\pi}_i$, therefore the above recursive equation is the same as

$$Z^{t+1} = (1-\overline{\pi}_i) A_i Z^t A_i' + Q_i \qquad (15)$$

$$Z^{t+1} = \tilde{A}_i Z^t \tilde{A}_i' + Q_i, \qquad (16)$$

where $\tilde{A}_i = \sqrt{1-\overline{\pi}_i} A_i$. Eqn. (16) is the discrete time Lyapunov equation. Since $(A_i, {Q_i}^{\frac{1}{2}})$ is controllable, so is $(\tilde{A}_i, {Q_i}^{\frac{1}{2}})$. If $\overline{\pi}_i < 1 - \frac{1}{{\alpha_i}^2}$, then $\tilde{A}_i$ is unstable, therefore the Lyapunov equation does not have a positive semidefinite fixed point to the Lyapunov equation (16).

From lemma 9 we know that $h_{\overline{\pi}_i}(Z^t)$ is a non-decreasing function, and it does not have a fixed point, thus it can be shown that $\lim_{t\to\infty} Z^t$ is unbounded for any $Z^0 \ge 0$.

Now since the recursion (14) initialized at any $Y^0 \ge 0$ yields $Y^{t_f} \ge 0$, therefore $\lim_{t\to\infty} Y^t$ is also unbounded for any $Y^0 \ge 0$.

Again from lemma 9(c), we know that if the recursive relation defined by MARE (11) and recursive eqn. (14) are initialized at the same $Y^0 = P_{i,0} \ge 0$, then

$$h_{\pi_i(t+1)}(Y^t) = \mathbb{E}[g_{\pi_i(t+1)}(P_{i,t})] = \mathbb{E}[P_{i,t+1}]$$

But limit of Y^{t+1} is unbounded, therefore there exists a $P_{i,0} \geq 0$, s.t. $\lim_{t\to\infty} \mathbb{E}[P_{i,t}]$ is unbounded.

Lemmas 8 and 10 tell us that if the sensor motion is modelled by an ergodic DTDS Markov chain, then the convergence properties of the MARE (11) can be completely determined by MARE (12) with the steady state probability $\overline{\pi}_i$.

Using the above facts and additional results from (Sinupoli *et al.*, 2003) we will derive conditions under which a single sensor fails to solve the dynamic coverage problem.

Definition 11. We say that the dynamic sensor coverage problem has been successfully solved if for any initial probability distribution of the sensors $\pi(0)$ the N limits

$$\lim_{t\to\infty} \mathbb{E}[P_{i,t}] \ , \ i \in \{1, 2, \cdots, N\}$$

are finite for any set of initial conditions $P_{i,0} \geq 0$.

If there exists an $i \in \{1, 2, \cdots, N\}$ such that $\lim_{t\to\infty} \mathbb{E}[P_{i,t}]$ is unbounded for some $P_{i,0} \geq 0$, then the sensors have failed to solve the dynamic coverage problem.

Theorem 12. Consider the system in eqn. (1). Let $(A_i, Q_i^{\frac{1}{2}})$ be controllable, (A_i, C_i) be detectable and A_i be unstable for all i. The sensor motion is governed by an ergodic DTDS Markov chain $\pi(t)$, s.t. $\lim_{t\to\infty} \pi(t) = \overline{\pi}$. Now if

$$\sum_{i=1}^{N} \frac{1}{\alpha_i^2} < N - 1, \tag{17}$$

where $\alpha_i = \max_j |\lambda_{i,j}|$, and $\lambda_{i,j}$ are the eigenvalues of A_i, then a single sensor fails to solve the dynamic coverage problem.

PROOF.

$$\sum_{i=1}^{N} \frac{1}{\alpha_i^2} < N - 1$$
$$\Rightarrow \sum_{i=1}^{N} \left(1 - \frac{1}{\alpha_i^2}\right) > 1 \tag{18}$$

Therefore for any steady state probability distribution $\overline{\pi}$ there exists an i s.t. $\overline{\pi}_i < 1 - 1/{\alpha_i}^2$. Now by lemma 10 we know that $\lim_{t\to\infty} \mathbb{E}[P_{i,t}]$ is unbounded for some initial condition $P_{i,0} \geq 0$. Thus a single sensor can not solve the dynamic sensor coverage problem.

It can be seen that eqn. (17) is a measure of how fast the systems evolve. In fig. 2 the region above

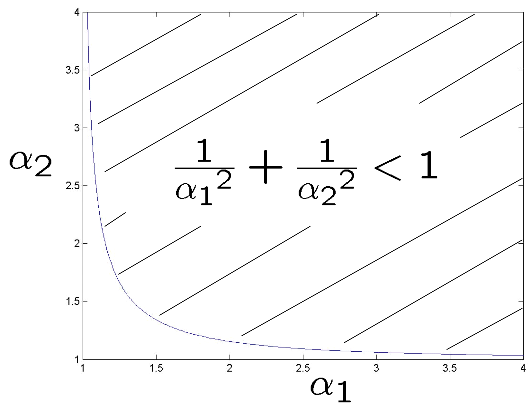

Fig. 2. Failure region

the curve is where a single sensor fails to solve the dynamic coverage problem for 2 systems. It should be noted that if one system is evolving very slowly then the sensor can tolerate very fast dynamics of the other system before it fails. In such a scenario the sensor distributes its time in such a way, that it spends relatively large amount of time observing the fast system.

We now give some conditions under which its possible to solve the dynamic sensor coverage problem by employing a single sensor. Before that we need to carry over a few terms from (Sinupoli *et al.*, 2003).

For real symmetric Y, define $\Psi_i(Y, Z)$ as

$$\Psi_i(Y, Z) = \begin{bmatrix} Y & \sqrt{\pi}(YA_i + ZC_i) & \sqrt{1-\pi}YA_i \\ \sqrt{\pi}(A_i'Y + C_i'Z') & Y & 0 \\ \sqrt{1-\pi}A_i'Y & 0 & Y \end{bmatrix} \tag{19}$$

and π_i^u

$$\pi_i^u = argmin_\pi [\exists 0 \leq Y \leq I, Z | \Psi_i(Y, Z) > 0] \tag{20}$$

Theorem 13. If $\sum_{i=1}^{N} \pi_i^u < 1$ then any sensor motion algorithm defined by an ergodic Markov chain $\pi_i(t)$ whose steady state probability vector $\overline{\pi}$ lies in the convex hull of the N points

$$\begin{bmatrix} 1 - \sum_{i\neq 1} \pi_i^u \\ \pi_2^u \\ \vdots \\ \vdots \\ \pi_N^u \end{bmatrix} \cdots \begin{bmatrix} \pi_1^u \\ \vdots \\ 1 - \sum_{i\neq j} \pi_i^u \\ \vdots \\ \pi_N^u \end{bmatrix} \cdots \begin{bmatrix} \pi_1^u \\ \pi_2^u \\ \vdots \\ \vdots \\ 1 - \sum_{i\neq N} \pi_i^u \end{bmatrix}$$

solves the dynamic coverage problem.

PROOF. Since $\overline{\pi}$ lies in the convex hull of the above points, therefore there exists $\beta_i \geq 0$, $\sum_i \beta_i = 1$, s.t.

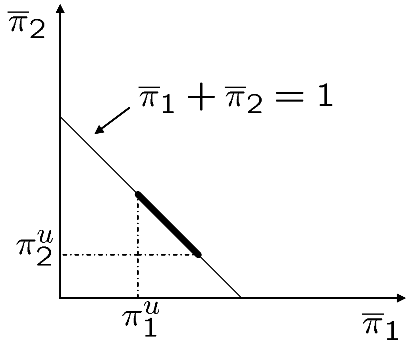

Fig. 3. Convex hull in two dimensions

$$\begin{aligned}\overline{\pi}_j &= \pi_j^u \sum_{i \neq j} \beta_i + \beta_j (1 - \sum_{i \neq j} \pi_i^u) \\ &= \pi_j^u (1 - \beta_j) + \beta_j (1 - \sum_{i \neq j} \pi_i^u) \\ &> \pi_j^u (1 - \beta_j) + \beta_j \pi_j^u \\ &= \pi_j^u\end{aligned}$$

Now it was shown in (Sinupoli *et al.*, 2003) that if $\overline{\pi_i} > \pi_i^u$ then the recursive relation defined by MARE (12)

$$Y^{t+1} = g_{\overline{\pi}_i}(Y^t)$$

is bounded for all initial conditions $Y^0 \geq 0$.

Therefore from lemma 8, we know that the recursive relation defined by

$$Z^{t+1} = g_{\pi_i(t+1)}(Z^t)$$

is bounded for all initial conditions $Z^0 \geq 0$.

$$\begin{aligned}\mathbb{E}(P_{i,t+1}) &= \mathbb{E}(g_{\pi_i(t+1)}(P_{i,t})) \\ &\leq g_{\pi_i(t+1)}(\mathbb{E}(P_{i,t}))\end{aligned}$$

by lemma 7

Now since $g_{\pi_i(t+1)}(\mathbb{E}(P_{i,t}))$ is bounded as $t \to \infty$ for all $\mathbb{E}(P_{i,0}) \geq 0$, therefore $\lim_{t\to\infty} \mathbb{E}(P_{i,t})$ is bounded.

For the two system case the convex hull is shown in fig. 3.

We are currently working on a geometric interpretation of the conditions in theorem 13, as we did for theorem 12. Our intuition says that the region where a single sensor can solve the dynamic coverage problem should lie somewhere in between the curve $1/{\alpha_1}^2 + 1/{\alpha_2}^2 = 1$ curve and the axes in fig. 2.

3. CONCLUSIONS AND FUTURE DIRECTIONS

In this paper we have used existing mathematical tools from the Markov chains and Kalman filter literature to define the dynamic sensor coverage problem. We have considered a simple case in which N spatially separated linear systems whose dynamics are decoupled have to be observed by a single mobile sensor. Due to the finite range of the sensor, it can make measurements for a particular system only if it happens to be at that system. We have modelled the motion of the sensor using an ergodic DTDS Markov chain. The conditions derived in the paper satisfy intuition.

There are several avenues of research that this paper opens up. The most immediate one is the construction of an appropriate transition probability matrix, once it is confirmed that a single sensor can solve the dynamic coverage problem. This will allow the sensor to make decisions on its next move based on its present location. One can also think of defining a cost function which can be used to trade off quality of estimates for overall sensor movement, thus reducing fuel consumption. One simple way to reduce sensor motion is to maximize the trace of the transition probability matrix. The matrix $\mathcal{T}$ can be sparse if movement between certain states is prohibited due to large separation in space.

Other research directions that we currently pursuing are solving the coverage problem, when the dynamics of the environment are coupled and dependent at different locations and multiple sensor case for non-cooperating and cooperating sensors.

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